MEASUREMENT

OF

ELECTRICAL RESISTANCE

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A TREATISE

ON

THE MEASUREMENT OF ELECTRICAL RESISTANCE

BY

WILLIAM ARTHUR PRICE, M.A., A.M.I.C.E.

FORMERLY SCHOLAR OF NEW COLLEGE, OXFORD

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PREFACE

The determination of resistance is the cardinal operation in electrical measurement, in the same sense that the comparison of masses is the cardinal operation in quantitative chemistry, or that of linear and angular distances in geodesy and surveying; and for similar reasons. The electrical resistance of many stable bodies is known to be a highly characteristic and permanent quality, and indeed is probably so of all; its measurement in terms of other resistances is performed with great ease: the exactness of the result does not depend on very accurate construction of the apparatus used: and this apparatus may be easily made of great sensibility. The measurement of resistance is little inferior, in point of exactness, to that of mass, and superior to that of angular or linear distance.

During some years the author has superintended the construction of a large number of resistance coils, and of various forms of instruments for their comparison and measurement, and he thinks that a systematic account of the methods and processes employed may be useful to the large and increasing body of electricians who use such apparatus.

All works on general electricity give some account of this subject, and a great deal of information is contained in the annual reports of the committee of the British Association on electrical standards. The mathematical investigation of the values of the currents and electromotive forces in Wheatstone's bridge was given by Mr. Oliver Heaviside; and a great number of papers on various points of interest have been published in scientific journals, and in the accounts of proceedings of scientific societies. The time at the author's disposal has not permitted of an exhaustive examination of these materials, but it is hoped that no points of importance in the ground covered by the book are unnoticed.

One very interesting branch of investigation is entirely omitted. The author has no practical experience of the determination of the unit of resistance in absolute measure, and any account he could give would be of inferior value.

Professor D. V. Jones finds that the values of very low resistances can be more exactly measured by their direct determinations in absolute measure by Lorenz' method, than by comparing them on some form of bridge with other resistances; but this process is not in extensive use at present. Should it come to be so, an account of Lorenz' method would be of essential importance to a work of this character.

The evidence recently obtained by Professors Dewar and Fleming that the electrical resistances of most pure metals vanish at the zero of the absolute thermodynamic scale, and are proportional to one another at

all temperatures, together with the work of Callender, indicates that a scale of temperatures based on the changes of the resistance of some pure metal, probably platinum, will take an important place in the art of thermometry. The resistance of a platinum wire is a quantity varying continuously with the temperature from the absolute zero, to temperatures above those of our steel and glass furnaces, and is readily measured at any point within a ten thousandth part of its value. The resistance of a pure platinum wire would be almost an ideal thermometric quantity. In the condition of purity now attainable it is of immense value.

4 Hardy Road,
Westcombe Park,
London.

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THE MEASUREMENT

OF

ELECTRICAL RESISTANCE

CHAPTER I.

The Definition and Law of Electrical Resistance, and its Unit of Measurement.

Article 1.] At every point of an homogeneous conductor in which are no currents of electricity the electrical potential is the same, but this is no longer the case if any currents are flowing. The potential is then distributed in such a way that the positive currents flow from places of higher to those of lower potential, and at right angles to the equipotential surfaces. For the complete discussion of such a system it is necessary to determine the relations between a current of electricity and the variation of the potential along its course. The simplest case to consider is that of a prismatic conductor whose length is great compared to its transverse dimensions, and whose ends are maintained at certain fixed potentials. In this case the currents of electricity flow parallel to the length of the conductor, and the potential is the same at every point of the area of any cross section perpendicular to the length. Hence the

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potential at any interior point of the conductor is the same as that of a point on the surface which lies in the same cross section, and can be readily ascertained.

This case, moreover, besides being the simplest in treatment, derives great practical importance from the extensive use made of long cylindrical conductors in telegraphy, and in the distribution of electric light and power. The following pages are confined to its discussion, and in speaking of a conductor we shall in future assume it to be prismatic, generally cylindrical. We require then to know the relation between the current flowing along any given prismatic conductor, and the differences between the potentials at the various points. To determine the relation between the flow of water along a pipe, and the head of water, or difference between the pressure at the ends, would be the analogous problem for an hydraulic engineer. Enunciated thus the electrical problem admits of a very simple answer—much simpler than that obtained for the hydraulic engineer. It depends upon an experimental result, called Ohm's law, which may be stated thus:—the difference of potential between any two points on a given homogeneous conductor measures directly the flow of electricity between them when that flow is steady; or, in other words, the ratio between the numerical values in any system of units of the difference of potential at the extremities of a homogeneous conductor, and the steady current flowing along it, is a constant quantity for that conductor.

This law as it stands gives no information as to the relative values of the flow, and the difference of potential, but states that if we can determine their numerical ratio for any given current, the same will hold good for any other current. This relation then between the fall of potential and the quantity of electrical flow is a characteristic property of the conductor, which, from its action

being analogous to that of a resisting or retarding force, is called the resistance of the conductor between the points where the potential is measured. Its inverse—viz. the ratio of the numerical value of the current to that of the difference of potential between two points—is called the conductivity of that section.

In Clerk Maxwell's words: 'In the first place the resistance of a conductor is independent of the strength of the current flowing through it.

'In the second place the resistance is independent of the electrical potential at which the current is maintained, and of the density of the distribution of electricity on the surface of the conductor.

'It depends entirely on the nature of the material of which the conductor is composed, on the state of aggregation of its parts, and its temperature.

'The resistance of a conductor may be measured to within one hundred thousandth part of its value, and so many conductors have now been tested that our assurance of the truth of Ohm's law is now very high.'

Ohm's law has been verified with a high degree of exactitude for some few materials, and with a less degree for a great many others; and no case of apparent failure has occurred within the author's knowledge for which an explanation cannot be found.

The determination of the resistance of conductors completely answers the problem stated in the opening paragraph, where the currents transmitted are steady: and in the more complex case of alternating or intermittent currents the resistance of the conductor is one of the most important quantities to be determined. Clerk Maxwell described it as being, in the then state of science, the cardinal operation in electricity, in the same sense that the determination of weight is the cardinal operation in chemistry.

2.] The electrical resistance of all metallic wires of like materials and dimensions and under similar physical circumstances is the same, and varies with changes of all these conditions.

It is different with different materials; e. g. the resistance of an iron wire is nearly seven times that of a silver wire of the same dimensions, that of manganin about twenty times, that of liquid mercury in a glass tube about sixty times.

The resistance varies with the dimensions; being in the case of the long prismatic conductors which we are considering, proportional to the length and inversely proportional to the sectional area.

The resistance also depends on the physical circumstances of the conductor, and especially on its temperature. The resistances of many pure solid metals, e. g. platinum, silver, copper, lead, are proportional to one another over a great range, and probably vanish at the absolute zero, but with alloys and liquid metals the law connecting the temperature with the resistance is very different. In almost every known case, however, the resistance of a metallic conductor increases with the temperature, manganin possessing the peculiar property of becoming a better conductor above a temperature of about 50° C. The resistance again of some hard drawn wires is reduced by annealing, there being a difference of about 3 per cent. in the resistances of hard drawn and annealed copper wires of the same gauge, and of about 6 per cent. with silver.

The specific resistances of non-metallic conductors are usually much higher, and diminish as the temperature rises, instead of increasing as in the case of metals.

Gases at ordinary pressures are perfect insulators, and, speaking generally, all conductors are opaque, and all insulators transparent. Electrolytic solutions and exhausted gases do not conduct electricity in the same way as conductors, but they convey electricity and possess real

resistance. This may be varied within wide limits with the strength of the solution, or the exhaustion of the gas. Their possessing resistance, however, does not necessarily subject them to Ohm's law.

3.] Suppose that we maintain the ends of a given conductor at potentials differing by E, and find that a current C is established. Then if we write $E = CR$, the physical quantity represented by R is called the resistance of the conductor, and Ohm's law asserts that R is a constant for that conductor, independent both of E and of C. The idea of a resistance can however, be expressed in the terms of ordinary mechanics. For multiplying both sides of the equation by C we have $EC = C^2R$. EC is the work done per unit of time by the source of electromotive force, and this work is converted into heat in the conductor. Then the rate at which work is done by a current C in heating a conductor whose resistance is R is C^2R, and the resistance may be defined as the ratio of the numerical values of the heat developed per unit of time and of the square of the current. In asserting then that R is a constant Ohm's law states that the rate at which work is done in heating the conductor is proportional to the square of the current passing[1].

4.] The application of Ohm's law has hitherto been limited to homogeneous conductors, but it is applicable to cases where points occur in the conductor at which an electromotive force is found either assisting or opposing the force applied from an external source; or, in other words, points at which work is absorbed or developed when a current passes through them. In this case Ohm's law is stated thus. The ratio to one another of the numerical values on any system of units of the current, and of the sum of the impressed electromotive force and the forces at

[1] This result was established experimentally by Joule, and is known as Joule's law.

the points in question taken with their proper signs, is a constant quantity, and is called the resistance of the conductor

E. g. suppose part of the circuit is a voltametric cell with platinum electrodes, which in a condition of rest has no electromotive force, but develops a back electromotive force of polarization when a current is passed through it from an external source. Then when the current is passing, the back electromotive force of the cell opposes the external force applied, and the effective force is the difference of the two. If E be the electromotive force applied, and e the back electromotive force of the cell at any instant, $E-e$ is the effective electromotive force of the circuit, and if $E-e = CR$, then Ohm's law asserts that R is independent of C. Multiplying both sides of the equation by C, $EC = eC + C^2R$. This equation states that EC, the total work done per unit of time by the external electromotive force is the sum of eC, the work done against the force e, and C^2R the work expended in heating the conductor in virtue of its resistance. The work eC done against e appears in the gaseous products of electrolysis, and, if these are not dissipated, the whole may be recovered by reversing the directions of the applied electromotive force and of the current. The external force is now assisted by that of the voltametric cell—due to the tendency of the gaseous products to recombine—and the current is increased thereby. When all the gases have recombined the cell is restored to its primitive condition, and then the whole of the work expended by the external source of energy has been employed in heating the conductor. Hence we can discriminate between the work done against the resistance, and that done against the electromotive force of the voltaic cell by the fact that the latter alone is recoverable on reversing the direction of the current.

In the case then where an independent electromotive force occurs in some part of the circuit, Ohm's law states that the electromotive force applied, increased or diminished by that at the critical point of the circuit, is proportional to the current: or alternatively when energy is absorbed or developed at some point of the circuit according to the direction of the current, then that part of the energy which is expended in heating the conductor, and is not recoverable on reversing the direction of the applied electromotive force, is proportional to the square of the current.

Conversely we find that the difference of the potentials at the ends of a conductor is not proportional to the current, nor is the total work expended per unit of time proportional to the square of the current, if there is any place at which a back electromotive force exists, or at which reversible work is absorbed or developed.

5.] Many different cases of such points are met with. Such for instance are the surfaces of the electrodes of any form of electrolytic cell, and the junctions of dissimilar metals. If the electrostatic or magnetic displacement surrounding a portion of a conductor be changing, that portion is a case in point. If two parts of a conductor of an uniform material are at different temperatures, or in different conditions of stress, some energy may be absorbed in passing a current from one part to the other which is recoverable on reversing the current. The first of these, viz. when two parts of the conductor are at different temperatures, is called the Thomson effect, having been discovered by Lord Kelvin. He showed that in copper a positive current from a hot point to a cooler one tends to cool the hot place and warm the cold one, while the action is reversed in iron. Heat is thus transferred with the current, apparently—though only apparently—in the same way as a stream of liquid would convey heat from a warm part of a pipe to a colder part. With a liquid

of course the effect is not reversible. In suggesting this analogy Lord Kelvin used the phrase 'specific heat of electricity.' A very curious case of the latter is that of a long copper wire suspended by its centre. Though the two parts hanging down have exactly the same resistance, yet if a current be passed up one and down the other, the potential at the point of suspension is less than the mean of those at the extremities. The explanation of this anomaly is found in the fact that a small amount of work is absorbed in driving a current from a part of the wire under small stress to a part under high stress, and this effect is reversed when the current passes in the opposite direction. Hence the ratio of the fall of potential to the current is greater for that part of the circuit where the current is ascending than for the part where it is descending: in fact it appears as if extra force were required to drive the current up hill, much as if electricity possessed weight[1].

The complete discussion of these effects is outside our subject, but it must not be forgotten that they are always liable to occur, and may seriously interfere with really exact measurements, especially of very low resistances. In every case with resistance coils heat will be absorbed or developed where the ends of the wire are soldered to the terminals, but since the two forces of contact act in opposite directions and tend to neutralize one another, they may be neglected if the two joints are at the same temperature. At the same time it must be remembered that in a coil whose ends are of different metal from the main wire, the current tends to cool one joint and heat the other, so that its passage might produce serious error by destroying the balance of the thermoelectric forces.

6.] A physical quantity is measured by comparing it with some other quantity of the same kind selected as

[1] The phenomenon is reversed in the case of an iron wire.—Shelford Bidwell, *Proc. Physical. Soc.* vol. ix. p. 3.

a standard. Thus intervals of time are measured by comparing them on a clock with the period of oscillation of a pendulum or balance-wheel, this period being determined in terms of that of the earth's rotation. Masses are measured by comparing on a balance the attractions of the earth on them and on other masses used as standards. So resistances of conductors are measured by comparison with other conductors which have been chosen as standards of reference.

Such measurements thus involve (*a*) the preparation of standards, (*b*) the discovery of some method or apparatus for comparing the two quantities.

The discussion of these two points is the subject of this book, and they will be dealt with in order. Describing first the points to be aimed at in the design of a resistance coil which will form a convenient standard of reference, the treatise proceeds to the various materials which have been used in the practical construction of resistance coils, and the mechanical processes and designs employed. The methods of comparing these coils with one another, and with other kinds of conductors, and the appliances used for the comparison, are then treated of. Detailed discussions of particular points of interest are placed in appendices.

7.] The principal points to be aimed at in the design and construction of a standard of resistance are the following.

i. The resistance of the standard must recover its original value whenever it is placed in the same external physical conditions. In practice this means that only those materials are available whose physical properties are unchanged by lapse of time, or by such variations of its circumstances as are likely to occur. Any substance liable to internal chemical change is quite unsuitable.

ii. The resistance must not be largely affected by variations in temperature, so that on placing it as nearly as possible in the same physical conditions as when

its value was originally determined, small errors in the adjustment of these conditions shall affect the value of the standard as little as possible. In practice this means that substances with a low temperature coefficient are to be preferred.

iii. In form and dimensions it must be convenient for use.

iv. To these may be added as a practical requirement for a form of resistance coil fit for extensive use, that it must be made and adjusted easily, and must not be too costly.

The relative importance of these conditions is different according as the standard is for practical everyday use or for occasional and exact reference. For the latter purpose the first condition is of absolute importance, and mercury — which fulfils it perfectly — may be the best substance to use, though it does not satisfy the other three conditions nearly as well as some solid alloys. For portable instruments, on the other hand, mercury would be quite inadmissible.

8.] We may briefly notice the question of the size of the unit. That originally taken as a convenient standard of reference was approximately the resistance of a mile of copper wire of a certain size. An exact value was assigned to it by Werner Siemens, who defined it as the resistance of a column of mercury 1 metre in length, and 1 square millimetre in section at the temperature of melting ice, and it has been called the Siemens unit. With the application of the C. G. S system to electrical measurement, and the adoption of the magnetic definitions, the unit of resistance was defined as the ratio of the centimetre to the second. The product of this quantity by 10^9, called the ohm, was designed for practical application—the C. G. S. unit being inconveniently small—and has nearly the same value as the Siemens unit. The exact determination of this unit and its practical realization has occupied many minds and many years. The original B. A. ohm of 1864 has been replaced

by more exact determinations of the C. G. S. unit. In 1884 the so-called legal ohm was agreed upon at the Paris Conference and defined as the resistance of a column of mercury 106 centimetres long, and of a sectional area of 1 square millimetre, measured at the temperature of melting ice. At the British Association meeting of 1892 the results of a large number of independent, and nearly concordant measurements of the C. G. S. unit were compared, and the following resolutions were passed with the concurrence of Dr. von Helmholtz representing the Curatorium of the Reichsanstalt.

i. That the resistance of a specified column of mercury be adopted as the practical unit of resistance.

ii. That 14·4521 grammes of mercury in the form of a column of uniform cross section 106·3 mm. in length at 0° C. be the specified column[1].

iii. That standards in mercury or solid metal having the same resistance as this column be made and deposited as standards of resistance for industrial purposes.

iv. That such standards be periodically compared with each other, and also that their values be redetermined at intervals in terms of a freshly set up mercury column.

The practical results arrived at after forty years' study are as follows.

The value of the C. G. S. unit has been determined by a number of electricians who have agreed within 6 parts in 10,000, and the mean value of their results has been adopted, and defined in terms of a column of mercury. Such a column of specified dimensions constitutes the primary standard.

The secondary standards are coils of wire of various

[1] This definition by its mass of the quantity of mercury is more direct than the definition of the sectional area of the tube, since this latter is only obtained by comparing the mass of the included mercury with the length of the column. It is equivalent to defining the cross section as having an area of 1 square millimetre.

alloys whose resistances are known to be very nearly constant, and which are much more convenient for use than the mercury column. Copies of these in more or less suitable materials and adjusted with all degrees of care are in common use.

TABLE OF THE RELATIVE VALUES OF THE UNITS OF RESISTANCE.

	In terms of the Siemens unit.	In terms of the B. A. unit of 1864.	In terms of the legal ohm.	In terms of the B. A. ohm of 1892.	In centimetres of mercury of 1 sq. mm. section at 0° C.
Siemens unit . .	1·	9535	·9434	·9407	100·
B. A. unit of 1864	1·0488	1·	·9894[1]	·9866	104·88
Legal ohm of 1884	1·06	1·0107[1]	1·	·9972	106
B. A. ohm of 1892	1·063	1·01358	1·0028	1·	106·3

[1] These figures, which differ from those usually given, are calculated from the figures of the British Association *Report* of 1892, for the ratio of the B. A. ohm of 1892 to the B. A. unit, and the defined lengths of the mercury columns for the B. A. ohm and the legal ohm respectively.

CHAPTER II.

The Materials used in the Construction of Standards of Resistance.

9.] The work of examining and testing different materials for the practical construction of standards of resistance was performed for the original committee of the British Association by Dr. Matthiessen and Mr. Hockin whose results are summarized below. The committee very quickly came to the conclusion that their choice lay between an insulated wire of some metal or alloy, and a glass tube of mercury, the former being certainly to be used for all practical work, the latter apparently admitting of repeated reconstruction. As a matter of fact the results obtained at that time by different experimenters, who attempted to determine the specific resistance of mercury by measuring the resistance of columns of the purest obtainable samples in long glass tubes, differed so much, that the committee placed little reliance on the method, and nearly all their experimental standards were constructed of metallic wires insulated with silk and paraffin wax.

The results of Dr. Matthiessen's experiments on the electrical resistance of metals and their alloys, performed before and after the appointment of the committee, were published in the *Philosophical Transactions of* 1860 *and* 1862, and in the British Association *Reports*. The following is a summary of them.

10.] Except in the cases of iron and thallium the resistance of a wire follows the same law of increase with temperature between 0° C. and 100° C. for all solid and pure metals. This law Dr. Matthiessen gave thus: calling the conductivity of the wire taken at 0° C. λ_0, and at t° C. λ_t

$$\lambda_t = \lambda_0 \{1 - \cdot 0037647t + \cdot 00000834t^2\}$$

the coefficients being calculated as the means of those obtained from the examination of ten different metals[1].

During recent years Professors Dewar and Fleming have experimented with the resistances of pure metals and alloys at low temperatures, and by the use of liquid oxygen and ethylene boiling under low pressures have obtained temperatures as low as 225° C. below zero. They have published accounts of their work in the *Philosophical Magazine*[2], and Figure 1 is a copy of the diagram attached to their paper of September, 1893, in which the general results are shown. It is reproduced here by the kind permission of the proprietors of that periodical.

In studying this diagram two points have to be borne in mind.

i. The scale of temperature used is based on equal changes of electrical resistance of a pure platinum wire, and the temperatures on that scale—called by Callendar platinum temperatures—are defined as follows: 'Let R_0 be the electrical resistance of a given wire of pure platinum at zero Centigrade, i. e. when immersed in melting ice, and let R_{100} be the resistance of the same wire at 100° C. Then an increment of temperature, which causes a change in the resistance of the platinum equal to $\frac{R_{100} - R_0}{100}$, is called one degree of platinum temperature.' These platinum temperatures are practically identical between 0° and 100° C. with those of the ordinary mercurial and air thermometric

[1] *Phil. Trans.* 1862, pp. 1–27.

[2] *Phil. Mag.* October, 1892, p. 327; and September, 1893, p. 271.

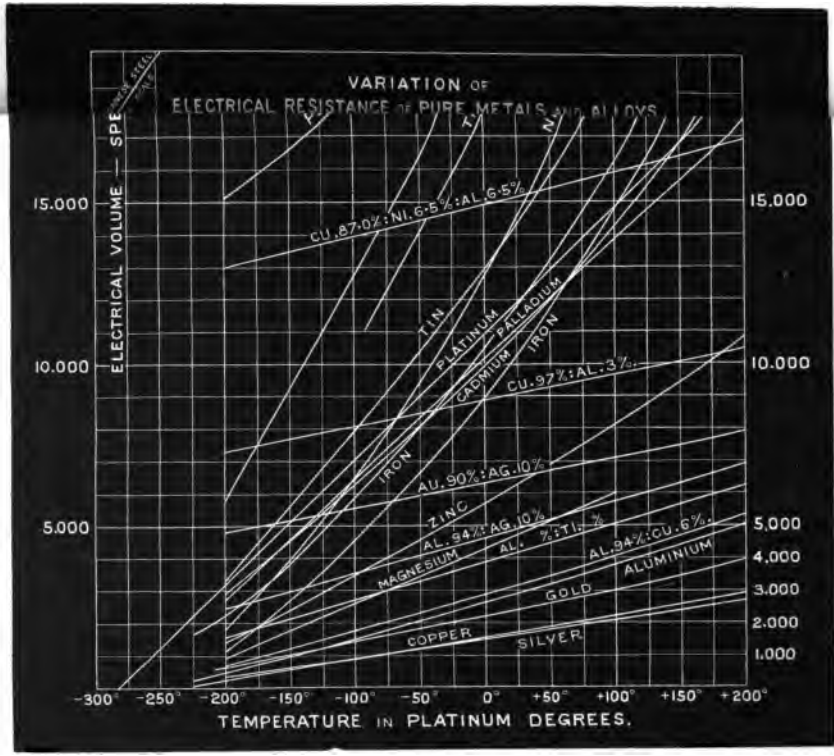


FIG. 1.

To face p. 14.

scales. Their values between 0° and 1000° C. have been examined by Callendar, who adopted the formula $e^{\frac{\cdot 003\,425\,9t}{1+\cdot 001\,529t}}$ as most nearly representing the ratio of the resistance of a wire of pure platinum at t° C. to its resistance at 0° C[1]. For low temperatures between −225° and 0°, the relation between the platinum temperatures and those of other scales has to be determined. We may remark, however, that the zero of the platinum scale shows a remarkable similarity to that obtained from the air thermometer. One of the two platinum wires which were examined indicated a zero, i. e. a temperature of zero resistance, at 283·4 platinum degrees below the temperature of melting ice, and the other 272·5 degrees below; the absolute zero of the air thermometer deduced from the expansion between 0° and 100° C. being −273° C.

ii. The resistances indicated in C. G S units are those measured between the opposite faces of a cube of the material, each side of which is one centimetre, measured when that material is at the temperature of melting ice. To obtain the volume specific resistance at any particular temperature, the data of the diagram would have to be corrected for the expansion or contraction of the material.

Bearing these points in mind the most striking features of the diagram are—

1. That the curves of the pure metals as well as of most of the alloys are nearly straight, the magnetic metals with their alloys and cadmium being the most marked exceptions. Thallium does not exhibit the peculiarity found by Matthiessen.

2. That all lines of pure metals tend more or less exactly to one point in the neighbourhood of the platinum zero,

[1] The use of a platinum wire in this way is due originally to the late Sir William Siemens, who employed it in his pyrometer for obtaining the temperatures of steel furnaces.

and that they do not diverge from this more than the line of the second sample of platinum referred to above would have done, had it been plotted on the diagram.

It appears then probable that pure metals would be perfect conductors near the zero of platinum temperature.

In view, moreover, of the behaviour of most alloys, it may be that such divergencies as are noticed in the lines of the nearly pure metals are due to small impurities, and that if metals could be obtained absolutely pure the lines would be straight, and concurrent at some point on the line of zero resistance. In that case the scale of temperatures obtained from the electrical resistance of all pure metals would be the same. This, however, is not established by the present results, nor is the zero of such a scale identified with that of the absolute thermodynamic scale.

11.] Matthiessen and Hockin found that the resistances of most metallic conductors continually rise with the temperature up to the melting-point. At this point—except in the cases of bismuth and antimony—there is a sudden rise in the resistance, which continues to increase afterwards, with the temperature, but more slowly than in the solid state. In the case of mercury the change of the conductivity of a portion enclosed in a glass tube was as follows,

$$\lambda_t = \lambda_0 \{1 - \cdot 0007443\,t - \cdot 0000008261\,t^2\};$$

this gives

$$R_t = R_0 \{1 + \cdot 0007443\,t + \cdot 00000138\,t^2 + \ldots + \ldots\}.$$

Two recent determinations by M. G. Guillaume, communicated to the British Association in 1892, give different values to these co-efficients,

$$(a)\ \mathrm{R}t = \mathrm{R}_0 \{1 + \cdot 00088023\,t + \cdot 0000010063\,t^2\}$$

and $$(b)\ \mathrm{R}t = \mathrm{R}_0 \{1 + \cdot 00088157\,t + \cdot 0000009909\,t^2\};$$

and for the specific resistance of mercury,

$$(a)\ \rho t = \rho_0 \{1 + \cdot 00088745\,t + \cdot 00000181\,t^2\}$$

$$(b)\ \rho t = \rho_0 \{1 + \cdot 00088876\,t + \cdot 0000010022\,t^2\},$$

the temperatures in M. Guillaume's experiments being reckoned from freezing-point by the air thermometer.

12.] iii. For the consideration of alloys Matthiessen found that metals could be divided into two sections—viz.

Section 1. Lead, Tin, Cadmium, and Zinc.

Section 2. Bismuth, Mercury, Antimony, Platinum, Palladium, Iron, Aluminium, Gold, Sodium, Copper, Silver, and probably most of the other metals;

and that alloys of two metals may be considered in three classes:

class *A* alloys of two metals both in the first section;

class *B* " " " both in the second section;

class *C* " " " one in the first section and the other in the second.

Class *A*. Both the specific gravity and the electrical conductivity of an alloy of this class may be calculated as if it was a simple mixture of its two constituents. Thus, if the specific gravities of the two metals be s_1 s_2, their specific conductivities k_1 k, and the respective volumes v_1 v_2, he found s the specific gravity of the alloy to be $\frac{s_1v_1+s_2v_2}{v_1+v_2}$, and k the specific conductivity to be $\frac{k_1v_1+k_2v_2}{v_1+v_2}$.

Neither of the other classes of alloys follows these laws.

The straight line Tin-Zinc in Fig. 2 graphically illustrates this law. The two abscissæ of any point on the line measured from the vertical boundary lines represent the percentage proportions by volume of the two constituents, and the ordinate of the point the specific conductivity of the alloy.

Class *B*. The conductivity of an alloy of two metals of the second section is generally much inferior to that of either of the constituents, and the law of its variation with the percentage composition much less simple. The conductivity alters but little through a considerable range of

different compositions, but rises very quickly as the percentage of either metal approaches 100. In other words, a very small amount of alloy increases enormously the resistance of an otherwise pure metal, while a large further addition may make but little more difference. This property is very well marked in an alloy of gold and silver; so much so indeed that Matthiessen proposed to use an alloy of two

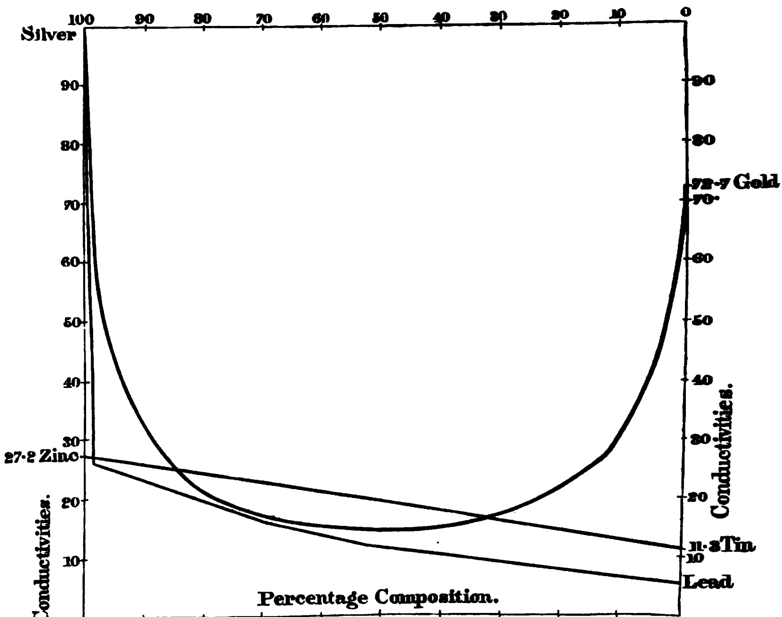


Fig. 2. Specific conductivities of alloys.

parts by weight of gold to one of silver for constructing standards of resistance, anticipating that the material could be obtained of very great electrical uniformity. This he expected because both metals could be obtained of great purity, while a small error in their exact proportions makes very little difference in the specific conductivity. The object gained, however, was of very small importance, and though the alloy was used in the construction of some of

the original B. A. standards, more reliance was placed on another of platinum and silver which Dr. Matthiessen afterwards discovered to possess greater advantages.

The line Silver-Gold in Fig. 2, extracted, like the others, from his paper in the *Philosophical Transactions*, shows the variation of conductivity with the percentage composition of an alloy of gold and silver. The sharp fall in the conductivity at each end of the curve caused by the first small amounts of alloy, with the comparatively small change in the middle of the curve, is well shown.

Matthiessen found further that the increase of resistance between 0° and 100° C. of alloys of class *B* is the same as that calculated from the mean of the components: thus we have the singular result that by compounding two metals of our second section we obtain an alloy whose specific resistance may be very much higher than that of either, or of their mean, while the increase of resistance between two temperatures remains the same as that of pure metals. Thus the coefficient of increase of resistance is reduced from the value for pure metals in the same proportion as the specific resistance is increased. To use Matthiessen's own words, 'the percentage decrement between 0° and 100° in the conducting power of an alloy, in a solid state, stands in the same ratio to the mean percentage decrement of the components between 0° and 100°, as the conducting power of the alloy at 100° does to the mean conducting power of the components at 100°.'

This discovery indicated a direction in which an alloy might be sought that would combine a high specific resistance with a low temperature coefficient; since any alloy of two of the metals of our second section which had a high specific resistance would necessarily have a low temperature coefficient.

Such an alloy was sought, and found, in a mixture of platinum and silver. After it had been determined that

its properties were not liable to secular change, this was practically adopted for making standard resistance coils, and was till recently the best known substance for the purpose.

Class *C*. Alloys of two metals, one taken from each section, have a conductivity which, while not generally inferior to that of both of the components, is yet less than the calculated mean. With alloys formed of a considerable percentage of the metal of the first section, and but little of the other, the conductivity is not much less than the mean; but when the alloy consists almost entirely of the second, and has only a small proportion of the first, we observe a strongly marked drop from the mean conductivity.

The line Silver-Lead in Fig. 2 is taken from Matthiessen's paper, and shows the properties of an alloy of this description.

As an illustration of the great effect of very small impurities on the conductivity of otherwise pure metals, the well-known case of copper may be referred to. This has been commercially produced of a conductivity 4 % higher than Matthiessen's purest specimens, to which—using them as a standard of conductivity—he assigned the number 100. The improvement has been effected by the elimination of very small impurities—principally arsenic—with greater perfection than he could attain at that time. It is worth noticing that he himself recognized that he had not employed sufficient care; he observed that pure metals could only be looked to for the construction of replaceable standards, if what he termed 'absolute care' were taken. He described his own as 'great care,' while a third or lower degree was named 'ordinary care.' The automatic distillation of nearly pure mercury in the vacuum of a Sprengel pump is probably 'absolutely' careful, but this process was unknown to Matthiessen.

13.] German silver—a three-metal alloy of copper, nickel,

and zinc—was well known at that time to possess a very low temperature coefficient, with a high specific resistance, but its electrical and physical qualities proved not to be permanent, and it was consequently not considered suitable for standard resistances. Its composition, however, led Matthiessen to look to the three-metal alloys as most likely to afford a substance superior to platinum silver, and though he did not succeed in finding one, his expectation has been justified by the recent introduction of manganin, a three-metal alloy containing no zinc. Indeed zinc is suspected to be the cause of the secular changes that take place in its alloys, such as brass and German silver.

A paragraph occurring in Matthiessen's Appendix B to the *British Association Report* of 1862, though not immediately bearing on our subject, may be noticed here. In this he points out that wires are occasionally defective through the production of internal cavities in the process of drawing, and describes how these are developed. His experience with copper and silver wires seems to anticipate on a small scale the Mannesmann process of rolling cavities inside solid ingots of steel for the purpose of making tubes. He writes as follows :—'It is well known that the wires of some metals require much more care in drawing than others: thus, copper and silver, if not annealed often enough during the process of drawing, will often become quite brittle, and break off short when bent. Now if the fracture be closely observed, it will be seen that the wire is hollow: in fact, wherever it is broken, cavities will be found, and sometimes of a millimetre or two in length: so that the wire may be almost regarded as a tube with a very fine bore. The reason of this is simply that in not annealing the wire often enough, the internal part of it becomes hard and brittle, whilst the outside remains annealed from the heat evolved by its passage through the holes of the drawplates: after a time however the inside, being very brittle, will give way, whilst the outside is still strong enough to bear the force used in drawing it through the drawplates. These places in the wire are easily discovered on drawing the wire finer; for then at these points the wire slightly collapses, owing to the quicker elongation of the weak points by the force used in drawing.'

Though, perhaps, this indicates the way in which the cavities of the Mannesmann tubes are first produced, viz. by working the solid metal in such a way as to keep the skin hot, it does not at all suggest the after development of the cavity which is a still more surprising phenomenon.

14.] Mercury appeared to offer great advantages, as no change can occur in its molecular structure or temper, and a standard defined as the resistance of a column of mercury of specified dimensions could apparently be replaced at any time if it were broken. Such a definition was urged upon the first B. A. committee by Kirchoff and W. Siemens, but determinations of the specific resistance of mercury made at that time by various observers showed so much discordance, that the committee decided—for the time at any rate—to prefer platinum-silver alloy. Since then, however, very concordant results have been obtained in the construction of mercury units, and it has now been decided—at least pro tem.—to define the standard of resistance as that of a column of mercury of certain specified mass and dimensions. The uniformity attained appears to be due to the improved methods of purifying mercury by distillation in vacuo.

15.] Guided by Dr. Matthiessen's investigations, the British Association committee reported, in 1866, that they had constructed ten standard resistances of five different materials, each representing the B. A. unit of resistance intended to be 10^9 C. G. S. units. These were made of the following materials:—

2 were coils of platinum wire,
2 ,, ,, gold-silver alloy (2 parts by weight of gold to 1 of silver),
2 ,, ,, platinum-iridium alloy,
2 ,, ,, platinum-silver alloy (2 parts by weight of platinum to 1 of silver),
2 were tubes of mercury.

These ten standards were equal to one another, and to the B. A. unit at temperatures lying between $14 \cdot 5°$ C. and $16 \cdot 5°$ C., and none of them differed more than ·03 per cent. from the value aimed at at $15 \cdot 5°$ C. Three parts in ten thousand may then be taken as the limit at that time of practical accuracy in the adjustment of coils of 1 ohm.

These ten standards, which were deposited first at Kew and afterwards at Cambridge, have remained in the hands of the original B. A. committee, and of another afterwards appointed, who have issued a number of copies of them in the platinum-silver alloy, and have compared and continue to compare with them other coils sent by the makers.

Other coils have been constructed since that time by the Committee representing later determinations of the ohm, but these do not offer any new features of material or construction.

The principal materials that have been commercially used for constructing resistances are the following:—

German silver,	Platinoid,
Platinum silver,	Manganin;

and for insulating materials, silk, paraffin wax, paraffin oil, and shellac. These will be described in order.

16.] *German silver* is an alloy of copper, nickel, and zinc. The proportions used in making wires for resistance coils are said to be—

4	parts by	weight of	copper,
2	„	„	nickel,
1	„	„	zinc.

The specific resistance is commonly stated as about thirteen times that of pure copper, the resistance between the opposite faces of a cube, each of whose edges is one centimetre in length, being 27 microhms, and its temperature coefficient as ·04 per cent. per degree Centigrade. The following is a general summary of the author's experience with this alloy.

German silver wires obtained in the ordinary way of commerce have given very different results. They differ greatly in hardness, to some extent in colour, and very greatly in their electrical properties. The harder and whiter varieties have a much higher specific resistance than the softer and yellower. The small sizes—say under

·01″—are commonly harder and more resilient than the larger, as though a different alloy were used for fine drawing; but larger sizes can be obtained very elastic and springy, and are largely used for the springs of telegraph keys.

In short lengths of these wires large mechanical differences may be observed. Thus, the author has seen lengths of helical German silver spring, of wire 36 mils. in diameter, some parts of which, when the whole is extended, will open widely, while in other parts the convolutions lie tight and close.

Uncovered wires hung in a store-room for some months have been found to have become quite brittle, and the same effect is often observed with brass wires. This happens more rapidly where gas is burnt, and is probably due to the presence of the zinc in the alloy. The same effect is produced by passing currents of electricity through them continually, strong enough to heat them well.

In one case, a number of resistance coils of German silver, insulated with silk and paraffin wax, were found after a few months to be so brittle that they broke up on the bobbins with the shocks of carriage; and on another occasion a resistance coil, which had been very carefully adjusted and had received a Cambridge certificate, was found to have dropped small pieces at the point where about two inches had been left bare near the soldered joint for adjustment. The author has no doubt that the cause in both these cases was insufficient protection from the air.

On the other hand, well-protected resistance coils of German silver wire, which had been in use in tropical countries for many years, have been found on examination to be as perfect as when they were first made.

In short, in no case where the metal has really been protected from the air by varnish or by a fair coat of paraffin wax has the author ever found a case of brittle or rotten

wire; and if proper precautions be observed in manufacture, German silver seems mechanically trustworthy.

The electrical properties differ very much in different commercial samples. Successive supplies of German silver wires from the same firm have varied in their specific electrical resistances in the ratio of 7 to 4; and the coil of a certain rheostat, which was designed to measure 22 ohms, gave only 14·5 ohms with one hank of wire, and 25 ohms with the next, the diameters and lengths of the wires being appreciably identical. As to the temperature coefficient; one sample of a very white and hard wire, said to be German silver, gave the same coefficient as platinoid: others have given much higher values.

In some cases the resistance of German silver has been found to change in process of time: this change seems inseparable from a zinc alloy, and renders it unsuitable for exact standards. The change, however, in well-made coils is usually very small, and quite within some other errors met with in practical work.

Probably German silver wires are manufactured with very varying degrees of care and attention, and though, when properly made, the alloy leaves little to be desired as the material for ordinary purposes, its reputation has suffered through the same name being applied to products of very different qualities.

17.] *Platinum Silver* is composed of two parts by weight of platinum to one part of silver. The colour is white. The material is very ductile and is readily drawn into fine wires. It is unoxidizable in air, and does not alloy with mercury. The conducting power at 0°C. is about one-fifteenth that of copper, its specific resistance—i. e. the resistance between two opposite faces of a cube, each edge of which is one centimetre—is 31 microhms. The resistance increases with temperature at the rate of ·031 per cent. per degree Centigrade. The electrical properties remain unaltered by

baking or annealing, and excepting some small changes, which are barely appreciable, the resistance of the coils made of this material in 1865 have remained constant.

Indeed, it would be an ideal substance for resistance coils were it not for its price, which is so high as to prohibit its use for ordinary purposes. Covered with silk it costs now from 45*s*. to 50*s*. per ounce.

18.] *Platinoid* is German silver with the addition of a small percentage of tungsten. The tungsten is added as a phosphide, a considerable percentage of which is, in the first place, fused with a portion of the copper. The nickel is then added, and then the zinc, and the remainder of the copper. The mixture requires to be re-fused more than once, and during the process the phosphorus, and a considerable portion of the tungsten, are removed as scoriæ. The alloy obtained is of a beautiful white colour, and has a remarkable power of resisting oxidation. The specific resistance is about 17 times that of copper, the resistance between the opposite faces of a cube, each edge of which is a centimetre, being 34 microhms. The percentage increase of resistance at 20° C. was found to be ·022 per degree [1].

19.] *Manganin.* Dr. St. Lindeck, in a valuable paper read before the British Association in 1892, gave an account of his experiments with this material, of which the following is an abstract:—

Mr. Weston of Newark, U.S.A., discovered that alloys containing manganese possess a very small temperature coefficient, and that it is even possible to obtain them with negative coefficients. The investigation of this matter was taken up at the Reichsanstalt at Charlottenburg, and an alloy, having the following composition, was decided on as most suitable for the purpose of making resistances: Copper 84 per cent., Manganese 12 per cent., Nickel about 4 per

[1] From the *Transactions of the Royal Society*, May 5, 1815. Paper by J. T. Bottomley.

cent. The annexed diagram (Fig. 3), taken from Dr. St. Lindeck's paper, shows the character of the resistance variation of manganin with temperature. The temperature coefficient is seen to be in any case extremely small, so that it may be neglected for almost all purposes. At a temperature about 45° C. the coefficient changes its sign and becomes negative.

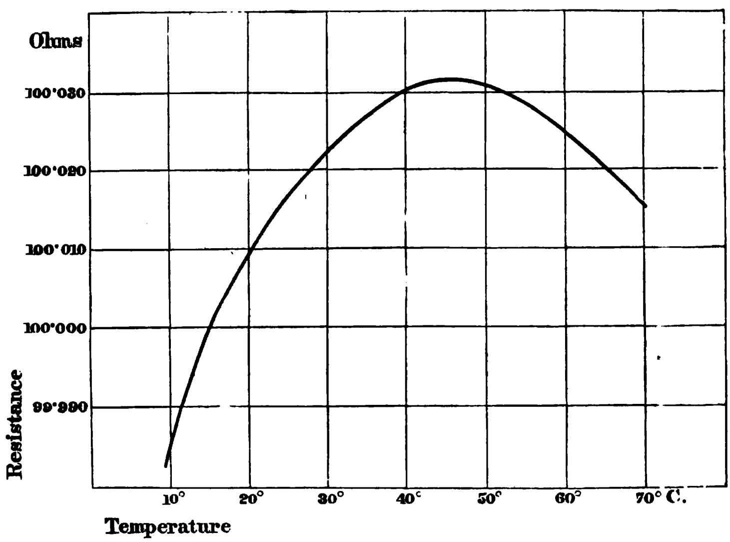


Fig. 3 Change of the resistance of a manganin wire with temperature.

The material is very soft, and can be drawn to the finest gauges, but it must not be annealed in free air because the manganese oxidizes very readily.

The bobbins after winding may be 'artificially aged[1]' by heating in an air bath at 140° for about five hours, the wire being heavily coated with shellac varnish. By this procedure we obtain very constant resistances; further, the shellac is melted at this temperature, and becomes after

[1] See p. 49.

cooling a hard, highly insulating mass, which protects the wire against any chemical action.

Dr. Lindeck also gave the thermoelectric force between manganin and copper as 1·5 microvolts per degree centigrade, against 20–30 microvolts for other alloys in common use. The minuteness of this coefficient is an advantage, if any difference in temperature between the two copper bars to which the ends of the coil are soldered is suspected.

The electrical properties of the material seem to be quite permanent, but long series of observations are required to establish this to the degree of certainty of the Platinum Silver standards which have been under examination since 1865.

The specific resistance is about 20·4 times that of copper, that of platinum silver being about 15 times.

20.] For the insulation of all these wires, only one substance has been practically used. Silk has been from the first the accepted insulator. Textile materials, from the great perfection to which the machinery for spinning and winding have been brought, as well as from their cheapness, offer great advantages for insulating wires. But all vegetable fibres and hairs are built up of very small cells or tubes, which tend to condense moisture, and cannot be permanently dried. No attempts to impregnate cotton or jute with insulating substances so perfectly as to completely fill the cells and prevent any moisture condensing there have been entirely successful, and, in the case of electric light cables insulated with impregnated jute, it is usual to draw a leaden tube over the jute to prevent the access of moist air. Silk, however, being formed by drawing a solid filament of gum from a reservoir in the head of the silk-worm—a filament which hardens on exposure to the air—contains no tubes or cells in which water may be condensed. It is liable of course, like every other body, to become moist on the surface, and its insu-

lating properties may become thereby injured. It is not, however, conspicuously hygroscopic, but to prevent any occurrence of damp it is usual to imbed the whole coil in a thick jacket of paraffin wax. Coils which are intended to be kept as standards in a testing-room or laboratory, and are not to be carried about, may be immersed permanently in a bath of paraffin oil. This oil being free from oxygen and sulphur, has no action on the wire, and experience has shown it to have no deteriorating effect on the silk. It has the advantages over paraffin wax that it allows the coil to be readily inspected, that the temperature can be quickly ascertained by stirring up the oil and immersing a thermometer, and, what may be more important still in the very highest class of standards, that it ensures that the wire is not under any stress such as might be caused by the solidification of paraffin wax or shellac. Strange's A. I. Crystal oil has been recommended for the purpose. Paraffin wax, moreover, has some tendency to absorb water, and the insulation of condensers, made with paraffined paper, soon deteriorates unless they are soldered up in air-tight cases. The wax retains permanently, however, a high insulating power, and any effect on the measured resistance of a coil of moderate dimensions would generally only be detected by very exact tests. Whether the coil is ultimately to be imbedded in wax or not, it is a good plan to dip the reel of insulated wire in melted wax before handling or winding. The insulating layer is less liable then to injury, or to pick up dirt: indeed, the Postal Telegraph engineers purchase their insulated wire for instruments already paraffined.

In some cases the author has used a thick coat of white hard varnish instead of the jacket of wax; it appears to answer well, and its mechanical strength and continuity offer some advantages over the wax, which is liable to crack and break off. So rarely however does it happen that the

coil itself is liable to be handled after mounting that this is rather a point to remember in case of its being required, than to suggest any departure from common practice.

For resistance coils, where the actual space occupied by the insulated wire is generally of small importance, two coverings of silk wound on in opposite directions are commonly used, and in important work three may be employed. The wire coverers work, however, with such evenness and precision that two coverings should be ample.

The use of shellac as the insulating substance for manganin wires has been described in Article 19 of this chapter.

CHAPTER III.

The Construction of the Spools or Bobbins.

21.] The construction of the bobbins on which the wires are to be wound is the next point to be considered, and the way in which the connexions are to be made to the coil is the most important factor in determining this. For a coil of first rate importance, to be used as a standard, it is necessary to make the bobbin in such a form that its temperature can be ascertained with great certainty, and that it can be connected to the comparing instruments with little handling, and with every certainty that no extra resistance is inserted at the joints. For the purpose of ascertaining its temperature the coils are usually permanently fixed in a metal box with long copper rods projecting from the top to which the ends of the wire are soldered. This box can then be placed in a bath of water or other liquid whose temperature can be taken with a thermometer, while connexions are made to the coil through the copper rods and mercury cups.

The annexed illustration (Fig. 4) shows the form of standard resistance coil devised for the first British Association committee, which was till recently the generally accepted form. It consists of a coil of wire on a metal bobbin with a tubular core, the ends being connected to a pair of thick copper rods, led through ebonite clamps, and bent downwards in the way shown, so as to be easily put into mercury cups. The whole coil is then slipped into

an outside case of thin sheet metal, in the form of two cylinders. The lower or smaller cylinder contains the wire coil, and the upper is filled with paraffin wax. The case up to the shoulder is intended to be placed in a bath of water, the temperature of which is taken with a thermometer placed in the central tube after the coil has been so long in the bath that the difference of temperature has been reduced to a very small amount.

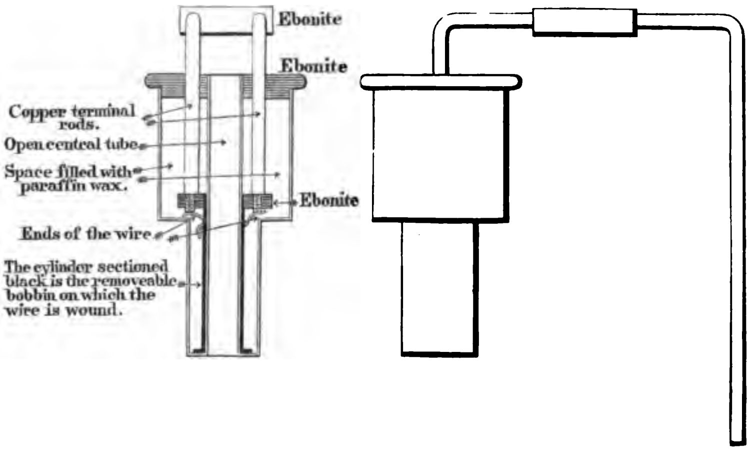


FIG. 4. British Association standard resistance coil.

Some years ago a new form of coil was proposed by Dr. Fleming, and has been adopted in a number of cases. It is shown in Fig. 5. The metal case of the wire coil consists of a hollow ring made in two halves and screwed together, in the hollow of which the wire is wound. Out of the upper half of the ring rise two sockets, into which are screwed long brass tubes which contain—insulated from them by ebonite—the copper terminal rods. The ebonite insulators, where they emerge from the tube, are formed with corrugations so as to insulate the copper rods from the outside tubes as effectively as possible. The advantage claimed for this form of coil over the older form is that

it offers greater facility for the water to circulate round the metal case, while the coil lies more nearly in one stratum of water than would the other. There is then less risk of two parts of the coil being in parts of the bath which are at different temperatures. Between the two tubes containing the terminal rods is seen a cylindrical attachment with a screw top. This provides a method of testing the tightness of the screwed-up joint by forcing air into the coil

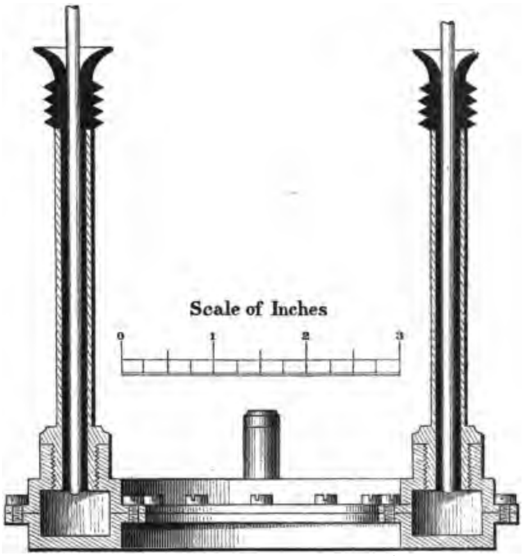


FIG. 5. **Fleming's form of standard resistance coil.**

through an india rubber tube when immersed in water, and observing whether bubbles of air pass through the joint anywhere. The copper terminal rods are of course bent over to reach the mercury cups as in the older form. The author ventures to think that a better plan than either of the above would be to put the coil in a thick copper case with a mercury recess for inserting a thermometer, and by placing the whole under a wooden shade to keep away draughts and dispense with the water bath altogether.

22.] For the construction and mounting of coils to be

used in ordinary testing practice, or in telegraph adjustments, the circumstances are somewhat different. It is quite clear that extreme accuracy in determining the temperature, and calculating the temperature coefficient, of each individual coil in a box containing a large set is not practicable, while some method of connecting it to, and disconnecting it from, the circuit must be devised which will occupy less space, and be more readily manipulated, than mercury cups. Considering also that a large number of coils may have to be placed in one box of small dimensions, it is clear that each coil must occupy as little space as is reasonably possible. Before determining how far we may sacrifice the advantages of the various points of construction of the standard form, it may be well to consider what degree of accuracy is attainable in the coils under these circumstances, and how far this will be affected by an altered construction.

We notice first that the change of resistance due to a change of temperature varies from 2 to 4·5 parts in 10,000 per degree C. according to the material used. Now, clearly, the exact temperature of a box of resistance coils thickly imbedded in paraffin wax, kept in a room whose temperature is subject to more or less rapid changes, and some coils of which, if previously used, may be heated by the passage of the testing current, is not readily ascertained within one or two degrees. Consequently the exact value of the coils, even if perfectly adjusted at some normal temperature, cannot be depended upon by some quantity of the order of say 5 parts in 10,000.

Again, the reports of the British Association Committee in recent years on a number of coils tested for, and presumably to be issued by the makers as standards, show in the majority of cases differences at the same temperature of from 1 to 4 parts in 10,000 among a set of equal coils adjusted by one manufacturer; and considerably larger

discrepancies among sets of coils intended to be multiples of one another. At the same time it is only fair to notice some sets of equal coils, apparently intended to be issued together to one customer as alternative standards, which have been adjusted to one value with an extraordinary degree of accuracy. Since the general average of these coils however has been presumably adjusted with especial care, we may safely conclude that the less important coils adjusted by the same manufacturers have, as a rule, at least as much error as this. In other words, that an error of this kind—i. e. some few parts in 10,000—is about the limit of practical adjustment. This last conclusion is borne out by the author's experience, who has found that if the work of adjustment is performed with system and care, coils can be brought by three or four adjustments within such an error. In testing with boxes containing sets of coils which are multiples of 1000 ohms, of 100, of 10, and of 1 ohm, it is clear that a much greater proportional error can be allowed on coils of a low resistance than on those of a high resistance; e.g. if an error of one part in 10,000 exists in a coil of 4000 ohms giving an error of ·4 ohm, an error of 1 per cent. in a coil of 1 ohm placed in series with the 4000 ohms will be of no importance whatever. Consequently it is not usually necessary in a box of this kind to pay nearly so much attention to the lower coils as to the higher ones, and the following limits may be assigned as the maximum errors permitted in a box fitted with the four sets of coils described above, and designed to measure with an accuracy of ·1 per cent.

In coils of—

1000	to	5000	ohms,	a maximum error	of	2	parts in	10,000
100	,,	500	,,	,,	,,	5	,,	10,000
10	,,	50	,,	,,	,,	7	,,	10,000
1	,,	5	,,	,,	,,	10	,,	10,000

As a matter of fact a considerable majority of the coils

actually adjusted will differ from their exact values by less than half of the above amounts, since, as will be explained later, the sizes of wires used for winding these coils make the adjustment of some few coils much more delicate than that of the others, and the assigned limits of maximum error will generally only be approached in the case of those few.

Comparing these errors of adjustment with the probable temperature errors estimated above we see that the latter will generally exceed the former.

Another source of error lies in the changes that take place in the specific resistance of the wire employed. In the case of German silver this sometimes reaches a figure of the same kind as the other errors, and will occasionally much exceed them. If, however, the wire be of good quality and carefully wound on the coils, it will not generally amount to any figure that can be detected, and is not to be expected.

In conclusion then, in using coils manufactured with due care, but excluding those which have been treated with special care to receive certificates, and for use as standards, we may expect to find errors in our results, due partly to defective adjustment, and partly to an uncertainty as to the exact temperature of the coils, which may together amount to from 5 to 10 parts in 10,000. Consequently, in testing resistances under such conditions as are met with in engineering and telegraph practice, a result stated in more than four figures will generally be misleading, and the fourth figure may be viewed with suspicion when the first is a large one.

Of course every degree of inaccuracy may occur between extreme error and the exactitude of the Cavendish laboratory, and the five-dial bridges found in instrument-makers' testing rooms are to be trusted to give five significant figures, but the above statement gives the general result that may be expected.

23.] So far in this discussion of the accuracy with which resistances may be measured we have dealt only with the standards of comparison, and not at all with the method of making that comparison. Practically some form of Wheatstone's bridge with a sensitive galvanometer is always employed, and this method is so good that no further error should be introduced, and the results will be precisely as accurate as the coils of the resistance box. That this would not necessarily be the case with an inferior mode of comparison may be seen from the parallel case of a foot rule, where, however accurately the rule is divided, measurements made with it by the method of laying an object against it are unlikely to be very good.

This degree of accuracy compares well with that of practical measurements of other kinds. Thus a balance which will weigh one ton within a pound, or a hundredweight within an ounce, a foot rule measuring correctly to one-hundredth of an inch, or sliding calipers measuring an inch within one-thousandth part, a method of measuring land giving results correct within five feet in a mile, or sextants with which three observers on a bridge agree within a minute of arc, would all be instances of good measurement. The marine chronometer and the astronomical clock have been brought to such perfection, and the rotation of the earth is so convenient a standard of reference, that measurements of time are in rather a different category to those of other quantities.

Electrical resistance then is readily measured in practical work with the same degree of accuracy as mass, length, and circular arc. In laboratory work its measurement is, however, somewhat less exact, as, with mass at least, quantities of convenient dimensions are comparable within one place in a million, electrical resistances only to one place in a hundred thousand.

24.] Recognizing then that, although single coils can be

brought into very close agreement with the standards, or that the temperatures at which they will agree with them can be very closely determined, yet in the case of sets of such coils mounted together in boxes any attempt at extreme accuracy would be futile and misleading, the makers of electrical apparatus have adopted various methods of mounting and connecting their resistance coils which offer advantages in compactness and simplicity at the expense of facility in making accurate comparisons with them. One method which has been commonly adopted is the following.

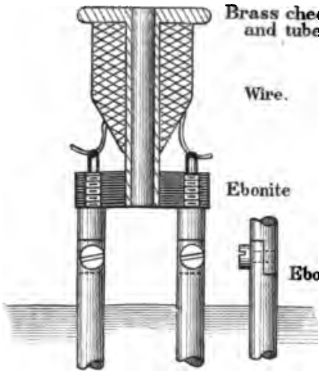


FIG. 6. Resistance coil on tubular bobbin.

The bobbin is formed of a small brass tube attached at one end to a circular brass disc, and at the other driven into a thicker disc of ebonite, the two discs forming the cheeks of the bobbin. Into the ebonite are screwed two studs whose ends pass through. The lower ends of the studs are stepped down so as to allow of their being attached to two small pillars of the same size set at the same distance apart; these pillars pass through an ebonite slab, and are screwed and soldered into the blocks or binding screws which form the external terminals of the coil. The joints between the studs and the pillars are made by small screws, and are also soldered to secure a good electrical contact.

The brass band of the bobbin is insulated by varnished paper, and the wire which occupies the space shown in the figure is soldered to the ends of the studs. The adjustment is made by changing the point at which the wire is soldered.

The advantages of this construction are that the whole circuit from one terminal block to the other is a continuous metallic conductor, every joint being soldered; and that the arrangement is very compact, no loose wires or ends being found outside the coils, and the whole coil on its bobbin being self-contained.

It has, however, several disadvantages. One is that the construction of the bobbin is rather complicated. The brass part of the bobbin is designed to assist in conducting away the heat developed in the wire by the testing current, but it would seem that when well insulated with varnished paper, and imbedded in paraffin wax, this action can only be very slight indeed; and, other things being equal, it seems better that the bobbin should be made of an insulating material than of a metal. Another disadvantage is that a considerable part of the bobbin is not available for carrying wire, as may be seen in the sketch.

Again, unless the wire be all in one length doubled back at the centre, there is necessarily a soldered joint in the inside of the coil where it cannot be inspected, and for practical reasons it is decidedly better, when any considerable length of wire is to be used, to wind a coil from two reels rather than to bend one piece and double it.

Lastly—and this in the author's opinion is a very serious objection—the coil cannot be placed in, or removed from, its position in its box without bringing a hot soldering bit near, not only to the coil itself, but also to all the other coils near it, a process which may affect their resistances.

25.] Another plan that is very commonly used, is to wind the wire upon a wooden bobbin and solder its outside ends

to two wires or spills soldered into the terminal blocks. The weight of the bobbin of wire is carried by another stout wire or screw. If the two inside ends are brought through the end of the bobbin and soldered together outside, all the objections to the other construction are met, but the arrangement is not sufficiently compact to be easily used in small boxes, and the coil cannot be removed without upsetting its adjustment.

26.] It is not easy to design a coil to meet all these difficulties, but a satisfactory compromise is obtained by abandoning the principle of a continuous soldered circuit. The plan adopted is the following: strong metal plates are permanently screwed to the bobbin to which are soldered the ends of the wire coil. The coil is secured in the box by clamping these between stout nuts, which both hold the coil and form the connexion to the blocks or terminals. To justify this plan two considerations are offered; firstly, that, excepting the mercury cups used in laboratory measurements, connexions and commutations in electrical testing are invariably made by clamping or pressing two clean metallic surfaces together, and one plan of doing this—viz. that of plugs—is implicitly relied on in all work of the kind. Secondly, with many thousands of coils connected in this way during several years of the author's practice, some sets of which have been removed for readjustment several times, no case of a bad connexion has occurred when the surfaces have been properly cleaned, and the nuts properly screwed up. In illustrations of coils (see Figs. 7, 28) made on this plan it will be seen that not only are two nuts, on a No. 2 brass screw, relied on for making the connexion, but the screw itself makes the connexion with the brass block by being firmly screwed into it. Since the screw is also the only means by which the block is held to the slab, the firmness of the block under the pressure of the plugs is a fair test of the tightness of the connexion.

Plates and nuts of the sizes illustrated are sufficient for coils as low as one-tenth of an ohm in boxes where such coils would be used as subdivisions added to others of a higher value; but in a case where coils of one-tenth or one-hundredth of an ohm are the highest in the set, it will generally be more economical to solder them up than to provide such large clamping surfaces and nuts as are in that case desirable.

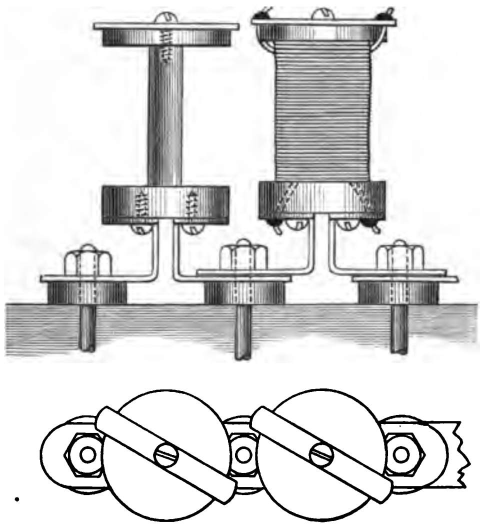


Fig. 7. Small resistance coils on wooden bobbins.

As an example of a form of coil made on this system Fig. 7 shows one suitable for many kinds of instruments, and especially for what is called a Post Office Resistance box. The boxwood bobbin is turned with one thick cheek and one thin one, the thick cheek being fitted with two brass plates of the form of Fig. 8. The holes *ac* in the upper part are for two small wood screws which secure it to the bobbin, and the hole *b* for the inside end of one wire which is brought through and soldered below. The oval hole in the lower part allows the coil to be used on screws of various

distances from one another. Screwed to the upper part is a smaller brass plate to which the outside ends of the wires are soldered, and which completes the circuit of the coil. It is at these latter soldered joints that the adjustments are made.

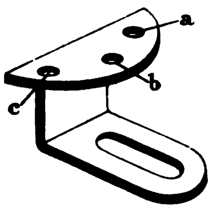


FIG. 8.

From this drawing it will be seen that each coil is self-contained, that it can be removed by loosening two nuts without the disturbance of any soldered connexion, and is as compact as the one described in Fig. 6. By employing the same stout screw to secure the block to its slab, and the coil to the block, room is obtained for a much larger screw than is possible when four screws are inserted in the same block. This last is mechanically a great advantage, since one large screw is much less troublesome to fit than four smaller ones, there are many fewer holes in the ebonite, and room is left in the block for small steady pins of the usual size, which answer the purpose of fixing the block much better than extra screws.

The construction of the coils used in very high resistances is described in Article 33.

CHAPTER IV.

THE WINDING OF RESISTANCE COILS.

27.] RESISTANCE coils are usually wound with two wires side by side, the ends of the two wires at one extremity being joined together, and the other ends forming the terminals of the whole resistance. By this means, when a current is passing along a wire, near every element of current in one direction is found an element of current in the other wire in an opposite direction. The actions of these currents at any point in the neighbourhood will be in opposite directions, and if the two elements are very close together the total effect is nearly zero; thus the inductance of the coil is reduced to a very small amount, and all disturbance of galvanometers or other magnetic apparatus is avoided.

The apparatus used for winding these two wires on at once is generally something like that shown in the illustration (Fig. 9). Through a support A is fixed a rod BB small enough to pass through the holes in any of the reels from which wire is to be drawn. On this rod are four collars, CCCC, having conical ends, two being fixed close to the support, the other two being moveable or fixed with set screws. The two reels put on the rod, one on each side of the standard, are secured by the moveable collars, so that they are free to turn on the cones, and will run truly on their axes. By setting up the moveable collars a roughly

adjustable friction is obtained sufficient to prevent the reels overrunning when drawing the wire off. The bobbin to be wound is shown at D mounted on a small lathe turned by hand. It is of the form of Figs. 7, 8, in which the two inside ends with which the winding is started are led through small holes at the point *b*, to be soldered up afterwards to

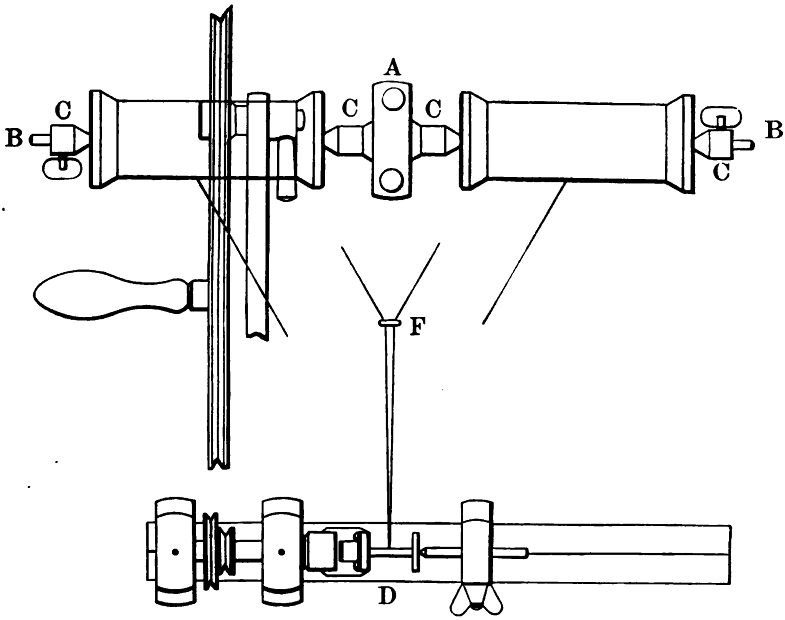


FIG. 9. Lathe for winding resistance coils.

the brass plates. An ordinary clockmaker's throw is a very convenient lathe for this purpose. Turning the lathe by hand is far more convenient than by foot, as it gives a much better command of the bobbin both for winding and for reversing the motion, and is especially valuable in winding coils which are not to be covered up, and whose outside layers have to be wound quite evenly. The two wires on leaving the reels may conveniently be led through a smooth eye at F.

Any one who is practically acquainted with the difficulty of filling a bobbin with fine wire so that each convolution lies evenly in its place, must appreciate the perfection of the machinery with which ordinary sewing cotton is spun and wound.

28.] The method just described of winding resistance coils doubly or differentially is usually quite sufficient to prevent any action on neighbouring instruments, since the currents employed are small, and there is no occasion to place the coils very near the other apparatus. But in the case of coils to be used as shunts close to a galvanometer, and of differentially wound galvanometers and relays for duplex telegraphy, the method is not exact enough. For in winding directly from two separate reels it is practically impossible to lay two fine wires close together: and though on the whole there will be as many current convolutions in one direction as in the other, yet the average or resultant current in one direction may lie nearer to one end of the coil, and that in the other direction lie nearer to the other end; and the whole coil will have a very appreciable inductance. This effect is often more considerable than would be supposed, the wire from the right hand reel having a tendency to lie continually to the right of that from the left hand reel, even though they are both passing through a small eye, while the irregularities of the surface of the partly wound coil tend to separate them still more widely. It is necessary then to use some means in these cases of winding the wire in such a way that each pair of convolutions shall lie closely together, and this is effectually done by twisting or stranding the two wires together before winding them on the coil. If this is done one may be certain that the two wires of one convolution are not separated by more than the thickness of their insulations, while if one such stranded convolution be viewed from an external point, parts first of one wire, then of the other

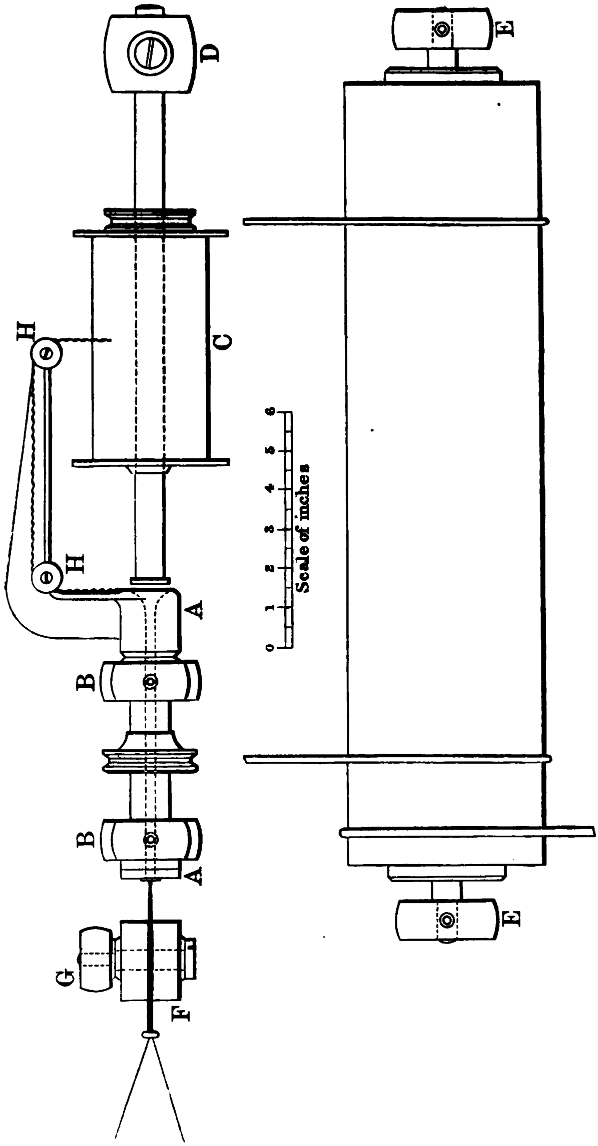


Fig. 10. Arrangement for laying up two wires.

alternately, as we look round the convolution, are the nearer to us. The total effect of the two currents will be more nearly zero than if they were merely circular and placed parallel to one another with the same distance between them. This principle of stranding or twisting the conductors is used in the telephone line to Paris, where the overhead wires between London and the English Channel are carried on the posts in such a way that they twist round one another with a lay equal to four times the interval between two posts.

The modified spinning-wheel shown in Fig. 10 has been arranged for laying up two or more wires for differential winding. The hollow mandrel A A carrying an overhanging arm revolves in bearings B B. The large metal bobbin C revolves on a fixed spindle carried by a support D. Both the mandrel and the bobbin are driven by bands from a drum shown below, which is carried in bearings E E, and is itself turned from the shafting or otherwise. The pulley on the mandrel is of slightly different diameter to that on the bobbin—in the drawing it is shown one-sixteenth larger—so that it only makes 16 revolutions while the bobbin makes 17. The wires drawn from reels, which may be conveniently carried in the way shown in Fig. 9, pass through a smooth eye, and each take one turn round a cylinder F covered with india rubber, and free to turn on a pin in the fixed support G. This plan ensures that exactly the same length of wire from each reel will be laid up, since neither wire can slip on the india rubber, and each turn of the cylinder F passes into the machine the same amount of each wire, viz. the length of its circumference. The two wires then pass together through the hollow mandrel, round the pulleys H H on the overhanging arm, and so are laid on to the bobbin. It is clear that every revolution of the mandrel twists the two wires once completely round one another, while in every 16 revolutions of

the mandrel, the bobbin, making 17 revolutions in the same direction, makes one complete turn under the arm, and draws off a length of the stranded wire equal to its circumference, which in the drawing is about $4\frac{3}{4}$ inches. Hence, in every turn of the stranded wire laid on the bobbin will be found 16 complete twists, or, in other words, the lay of the strand is a little more than $\frac{1}{4}$ inch.

29.] The two wires having been wound on the bobbin, the insulation of the two from one another may next be tested, since, if this is not perfect, two paths are established for the current from one terminal to the other, one along the wire, the other through the insulation. This should always be done with the differentially wound coils of wire of relays and galvanometers which are only once covered with silk, and with resistance coils carrying great lengths of wire even though this is doubly or trebly covered. In the former case, though small changes of resistance are of no importance, a low insulation may easily develop into a contact between the wires. In the second case a low insulation may lower the resistance to a serious extent.

If, however, the insulated wires have been kept in a warm, dry place, and care and cleanliness observed in winding the coils, the insulation resistance should in the case of double silk covered wire be very high indeed. In one case of 40 coils wound to 25,000 ohms each, prepared for a resistance of 1 megohm, the apparatus for testing did not indicate any leakage at all, showing the insulation of each one to be at least as high as 150 megohms. This being quite sufficient, no further test was taken. In another case where the wires had been stored in a place that was not sufficiently dry, similar coils gave insulation resistances between 20 and 30 megohms, which would reduce the resistance of the wire circuit by one two thousandth part. This defect would not matter if it could be trusted to remain constant, since the wire circuit would be adjusted to a higher resistance to

compensate for it: insulation resistances of this kind, however, cannot be trusted to keep constant.

In the case of coils of less than 1000 feet of double covered wire no insulation test should be necessary. unless previous experience, or uncertainty as to the quality of the insulated covering. should suggest its being done, but the method of constructing resistance coils described in this book always allows the test to be taken after winding, and in cases of coils of a greater resistance than 10,000 ohms it may be done after adjustment. After the resistance of the coil has been roughly adjusted by cutting off the wires nearly to the point required, the ends may be soldered to the brass plates, and it is a good plan to give the whole a coat of paraffin wax immediately afterwards.

30.] The coils must be left alone for some days after winding. A curious property of German silver wires may be noticed. After winding, after heating by soldering, and after any but the gentlest handling, the resistance of the wire rises slightly, and takes some time to recover its proper value. About three days seem to be required after winding, and twenty-four hours is sufficient after testing and adjustment by resoldering, but small changes are sometimes observed for long afterwards.

This gradual change after winding has been called the 'ageing' of the coil. It may be accomplished in a short time by keeping the coil at a high temperature for some hours after winding, a process that was used by Matthiessen, and again, lately, with the coils made at the Reichsanstalt at Charlottenburg. The coils there are heated for five hours consecutively in an air bath at 140 C., and are said to be 'artificially aged.'

The adjustment is made after careful comparison with standard coils on some form of Wheatstone's bridge, by unsoldering one of the ends, and resoldering it again at another point of the wire distant from the last by the

amount of the correction to be made. The coil should then be laid aside till the next day, and then tested and corrected again. This is repeated till the coil is correct. An expert tester dealing with a large number of coils at once will test and adjust them very rapidly and with great precision, the whole of the testing being done first, and the correcting afterwards. The resistance of each coil is entered in a book from its daily test, and, as soon as it is found to be within the limits of error allowed, is ticked off and laid on one side, to be finally tested again before being mounted in its box. Three or four tests and adjustments are commonly enough to bring coils within the limits of error given in article 22.

31.] For soldering the wires resin is the best flux known, and is commonly given in specifications. Chloride of zinc acts much more easily, but will continue to corrode any metal containing copper for long afterwards, and is also very liable to destroy the insulation of the ebonite surface. It should on no account be used. Experts with the soldering bit prefer it to any other tool, and its universal use by tinsmiths shows its value; but the author's hand finds it a very clumsy weapon, and a small blow-pipe flame seems to fuse the joint more rapidly, and heat the rest of the coil less.

With wires of small diameters used for considerable resistances and corrected within the limits proposed in article 22, the distance by which the soldered joint has to be moved along the wire in making the final adjustments is generally very appreciable—thus the final adjustment on a coil of 1000 ohms of platinoid wire of a diameter ·004″ is about $\frac{1}{4}$″. With the short lengths of thicker wire, however, used for small resistances, the alteration in the position of the soldered point is so small, and the exact point where the soldered joint is to be taken so indefinite owing to the size of the joint, that it is difficult to make a fine adjustment. In these

cases two methods may be used. The first is to solder the wire at such a point that its resistance is a little too low, and then to raise the resistance by reducing the section of the wire with a file near the joint. Another and much better method is to leave the resistance of the wire too high, and to reduce it by soldering across its terminal plates a longer wire of high resistance to act as a shunt. The illustration (Fig. 11) shows a coil of thick wire shunted by a thin one. In this case the thick wire may be cut off in one length and soldered to the brass plates, leaving the third brass plate at the top of the bobbin for the adjustment of the fine wire shunt, whose resistance can be altered without touching the points where the main wire is soldered up. A convenient way of determining what is to be the resistance of the fine wire is to connect a resistance box in parallel with the coil to be adjusted on the Wheatstone's bridge, and alter the resistance open in the box till the standard coil is balanced.

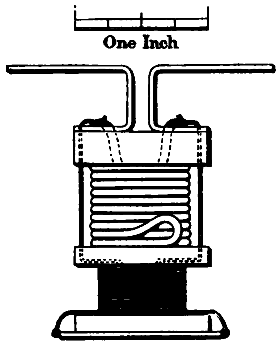


FIG. 11. A thick wire coil shunted by a fine wire.

32.] The sizes of the wires to be used in a given set of coils is determined mainly by considerations of convenient adjustment. It is necessary to provide wires of such a diameter that the smallest length by which the final adjustment of resistance is to be made shall be long enough for the purpose.

The following table gives the sizes of platinoid wires which may be conveniently used for coils of the size of Fig. 7. The first column gives the resistance of the coil, the second column the size of the wire, the third column the maximum error allowed under the rule of article 22 for this class of work, and the fourth column gives the length of the wire which corresponds to that error, and which is

consequently the smallest amount by which the soldered joint can require to be changed.

Resistance of coil.	Diameter of wire.	Maximum error allowed.	Corresponding length of wire.
1 ohm	48 mils	·001 ohm	·12 inch.
2 „	48 „	·002 „	·24 „
3 „	36 „	·003 „	·22 „
4 „	36 „	·004 „	·29 „
10 „	24 „	·007 „	·22 „
20 „	18 „	·014 „	·36 „
30 „	14 „	·021 „	·34 „
40 „	12 „	·028 „	·24 „
100 „	10 „	·05 „	·28 „
200 „	8 „	·1 „	·36 „
300 „	7 „	·15 „	·4 „
400 „	5 „	·2 „	·28 „
1000 „	5 „	·2 „	·28 „
2000 „	4 „	·4 „	·36 „
3000 „	4 „	·6 „	·54 „
4000 „	4 „	·8 „	·72 „

In this arrangement it is seen that no adjustment has to be made of less than $\frac{1}{8}$ inch in any coil excepting that of 1 ohm and this may be adjusted if necessary by the shunt method described in article 31. The smallest size of wire used is ·004 inch. Finer wires than this, unless economy of space is very important are not to be recommended. The largest size, ·048 inch. is as large as can be conveniently coiled on these small bobbins.

The bobbins shown in Fig. 28, and used in boxes designed for measurements of a higher degree of accuracy, have three times the capacity and superficial area of those we have been discussing and carry larger sizes of wire. The thicker wire offers better facilities for close adjustment, and the larger surface radiates heat more rapidly, and the temperature of the coil rises less with the testing current.

The sizes of wire used may be as follows:

For coils of	·1 ohm,	wire of	·048 inch diameter
„	1 „	„	·048 „ „

these coils being generally adjusted by one of the methods of article 31.

For coils of	10 ohms,	wire of	·036	inch	diameter.
,,	100	,,	·014	,,	,,
,,	1000	,,	·0065	,,	,,
,,	10000	,,	·004	,,	,,

For boxes whose coils are wound with platinum silver wires, and in which the cost of the wire is a serious item, the following sizes are commonly found:

For 1	ohm coils, wire of	24	mils.	For 100	ohm coils, wire of	6	mils.
2	,,	20	,,	200	,,	5	,,
3	,,	20	,,	300	,,	5	,,
4	,,	16	,,	400	,,	4	,,
10	,,	12	,,	1000	,,	3	,,
20	,,	10	,,	2000	,,	3	,,
30	,,	10	,,	3000	,,	3	,,
40	,,	8	,,	4000	,,	3	,,

32 a.] Questions as to the amount of current which the coil will carry without undue heating, or as to its temperature when conveying a steady current, are dealt with by the following considerations.

A coil carrying a steady current of electricity gradually rises to such a temperature that heat is then thrown off by radiation and conduction at the same rate as it is developed by the current. The second of these quantities is readily determined. Suppose R to be the resistance of the coil in ohms, C the current passing in ampères, and E the electromotive force applied to the terminals of the coil in volts, so that $E = CR$; then heat is developed in the coil at the rate of EC watts and $EC = C^2R = \frac{E^2}{R}$.

If the current be a fixed quantity, the heat developed per second is proportional to the resistance. Thus, if a battery of high resistance be connected in succession to a number of coils of low resistance, so that the current is appreciably the same in all, heat will be developed in them in proportion to their resistances. Suppose a Daniell battery of 10·7 volts and 100 ohms to be connected successively to coils of

1, 2, 3, 4 ohms. Heat will be developed in them at the rates of ·0112 watt, ·022 watt, ·0324 watt, ·0424 watt, quantities which are appreciably proportional to the resistances. The resistance of the coil in which the current from the above battery would develop most heat is the same as that of the battery, viz. 100 ohms, and the rate of development would be ·285 watt. The 100 ohm coils would consequently be the weakest of the set, so far as their power of resisting the heating action of such a battery.

If the same electromotive force be applied to a series of coils, the heat developed per second in any coil is inversely proportional to its resistance. Thus, if a Leclanché battery of 8 volts and small resistance is connected in succession to the coils of 1, 2, 3, 4 ohms, the heat developed will be at the rate of 64 watts, 32 watts, 21·3 watts, 16 watts, quantities inversely proportional to the resistances. In this case the 1 ohm coil would be the weakest of the set.

The rate at which heat will be dissipated by any particular coil can only be determined exactly by experiment, but it may be assumed for coils under similar conditions to be proportional to the superficial area of the coil, and to the difference of temperature between it and the surrounding air. Professor S. P. Thompson gives as a working rule for the large coils of electromagnets that 1 square inch of surface warmed 1° Faht. above the surrounding air will emit heat at the rate of $\frac{1}{255}$ watt[1], and the author's experiments have shown this rule to hold very fairly well for resistance coils covered with paraffin wax and connected up in a resistance box.

Coils made in the form of Fig. 7 have a superficial area of from 4–6 square inches according to the amount of wire wound on them, and assuming the above rule to hold, heat may be developed at the rate of ·08 watt to ·12 watt without

[1] *The Electromagnet*, Silvanus P. Thompson, p. 193.

heating them more than 5° Faht., i.e. without increasing the resistance of platinoid by more than 6 parts in 10,000. If the battery power used with such coils be limited by this very safe rule, the very short duration of the currents employed in bridge testing will reduce the change of resistance to a negligible amount. In the coils of the size of Fig. 28 ·3 watt may probably be developed with the same rise of temperature.

No advantage, as regards power of carrying current, is gained by winding a coil of given resistance with a thick wire, as compared with a thin one, except so far as greater radiating surface is obtained or so far as the heat is developed in a greater mass of material, so that the steady current can be continued longer before a given rise of temperature is produced.

33.] For constructing such very high standards of resistance as are used in testing the insulation of cables several substances have been used; pencil lines drawn on ground glass or wood; selenium; cells filled with a mixture of two powders, one conducting, the other non-conducting; pencils made of a mixture of graphite and Stourbridge clay; a 10 per cent solution of cadmium iodide in amylic alcohol—Hittorf's solution; but a resistance of fine wire is much the most trustworthy. The most convenient resistance is either a megohm, or one-tenth of a megohm, subdivided into several sections so that a lower resistance can be used, if it is wanted.

34.] A resistance box of 1 megohm is conveniently constructed of 40 coils of 25,000 ohms each, and connected up into 4 sections of 250,000 ohms each. Each coil is wound upon a separate box-wood bobbin of the form shown in Fig. 12, and consists of two wires of 4 mils. diameter, covered with two layers of paraffined silk. The bobbin has four small brass plates attached to it, two on each cheek; to those on one cheek, *a* and *b*, are soldered

the inside ends of the wire, and the outside ends to the two plates, *a* and *b* on the other cheek. The inside ends are led from the plates to the barrel of the bobbin, where the winding begins, along grooves cut in the wood, so that the other layers are not pressed against those parts. *a*, *a* are the terminals of one wire; *b*, *b*, those of the other. The barrel of the bobbin is of considerable diameter to reduce the pressure upon the inner layers of wire. There are fewer layers required to obtain the requisite resistance than would be the case if the barrel were small, and each turn being of a larger radius, exerts less pressure, since, with a given tension, the pressure is inversely proportional to the radius. The wires are laid on together, but not stranded. Before the resistance is adjusted the insulation resistance between the two wires is measured, a test which would not be possible if the inside ends were permanently joined up at the barrel. The resistance between the two wires should not be less than 100 megohms, and may well be much higher. When the adjustment has been completed the coils are ready for mounting in a box. This may be done in any way that is convenient, one point however being called attention to. Consider the two ways shown in Fig. 13 of connecting ten of these coils in series. In the first arrangement the current from A to B passes through one wire of each coil, and is returned to C through the other wires. In the second arrangement the current from *a* to *b* is carried through both wires of any coil before passing to the next, and

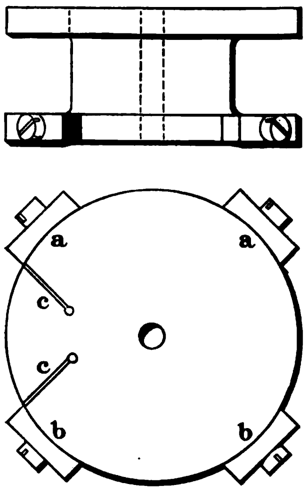


FIG. 12. Bobbin used for very long wires.

returned to *c* through an independent lead. The two arrangements are equivalent as far as the resistance of the metallic conductor is considered, but the second is the best for two reasons connected with the insulation. Suppose the dielectric resistance between the two wires of each coil of 25,000 ohms to be 100 megohms. In the first arrangement the dielectric resistance between the whole conductor A B, which includes one wire from each coil, and the return B C, which includes the others, is one-tenth of that resistance, i. e. 10 megohms. The direct measurement

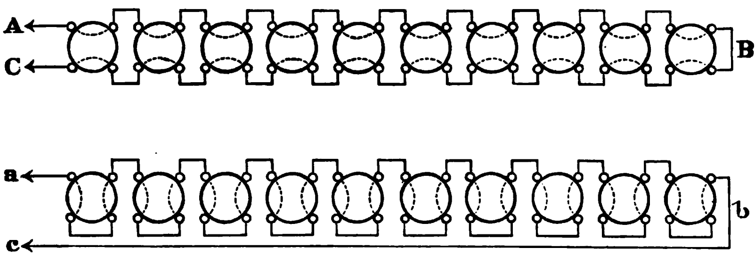


Fig. 13.

of this quantity can of course be made with the connexion at B broken, so that the testing-current from A to C passes only through the insulation. The conductor resistance of the ten coils in series is 250,000 ohms. Now suppose a battery of 1 volt to be connected to A C, the connexion at B being made. The current through the wire is 4 microampères, and the current through the insulation is $\frac{1}{20}$ microampère. If the connexion at B were broken, the current through the dielectric would be $\frac{1}{10}$ microampère, but this connexion reduces by one-half the average difference of potential between adjacent parts of the going and returning circuits. The current then from A to C is 4·05 microampères, and the effective resistance of the coils is reduced by one part in eighty With the second arrangement the electromotive force at the terminals of each coil is $\frac{1}{10}$ volt, the current through the con-

ductor 4 microampères, and that through the insulation is $\frac{1}{2000}$ microampère, being one-half of its value if the two wires were disconnected. The current through the coil is 4·0005 microampères, and the effective resistance of each coil is reduced by 1 part in 8000. Thus, if the ten coils be connected as in the first arrangement, the imperfect insulation reduces the conductor resistance by one-eightieth. The same coils will lose only one eight-thousandth part if connected in the second way.

By the second arrangement no two adjacent wires are at potentials differing by more than one-tenth of the whole electromotive force applied to the terminals AB, while in the first the difference of potentials between the wires when they approach AB would be the whole of the applied force. When large battery powers of 900 volts and upwards are used to test cables this is a matter of considerable importance.

CHAPTER V.

On Commutators, Switches, and Connexions.

35.] In the use of all electrical apparatus, and especially of adjustable resistances, continual changes have to be made in the connexions of the various parts, both in arranging the tests, and in the actual operation of them. A great number of arrangements have been devised at different times for making these changes more or less readily. and the circumstances of each particular case must determine which arrangement is to be preferred. They all aim at providing a sufficient metallic contact when the switch or commutator is closed, and of allowing that contact to be made or broken easily when required. What constitutes a sufficient contact depends upon circumstances, such as the amount of current to be passed, and the degree of importance of resistance at the point of contact. The switches are of two kinds: they either depend on the use of mercury, or on the pressure together of two clean solid metallic surfaces.

36.] The use of mercury is universally employed in laboratories for making accurate measurements of resistance, because with a little care connexions can be made between the parts of apparatus with certainty that no resistance is introduced at the joints. It is commonly used in what are called mercury cups. These are small vessels containing mercury into which dip the amalgamated ends of copper rods between which electrical connexion is to be

made. Fig. 14 shows a convenient form described in the report of the British Association committee for 1865 which requires a very small amount of mercury. Each consists of a cylindrical cup of box-wood containing a disc of copper at the bottom, and above it a cylinder of box-wood pierced with two holes somewhat larger than the copper rods used for the terminals of the coils, and for other connexions.

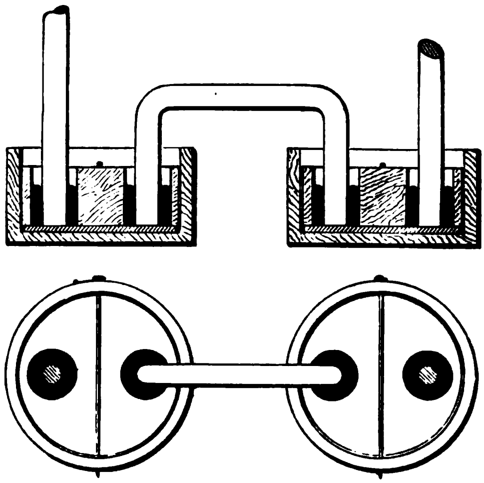


Fig 14. Mercury cups.

By well amalgamating the copper disc and the lower ends of the copper rods, and putting a small amount of mercury in the cup, a very secure connexion is made between two rods inserted. The rods should be held down in some way by springs or clamps so as to press on the copper disc. The box-wood cylinder is prevented from rising out of the cup under the upward pressure of the mercury by a small pin thrust through the cylinder and the sides. The illustration shows the form of the cup and of the connexion piece used to connect two such cups. Both the ends of the copper rods and the copper disc should be freshly amalgamated before use whenever the mercury cup

has been emptied. This may readily be done by well cleaning the surface with emery paper and then rubbing it with an aqueous solution of nitrate or chloride of mercury. The mercury in the solution being replaced by copper, is deposited on the surface of the rod, which is thus slightly amalgamated, and will readily pick up more mercury when dipped into it.

37.] Another well-known case in which mercury is used for making an electrical connexion is that of the Siemens dynamometer. Here the current, which may amount to as much as 1000 ampères, is led into the moveable circuit of the apparatus through two vessels full of mercury placed one below the other in the axis of the moving part (Fig. 15). A swinging frame of this kind had been previously used by Ampère in his experiments on the mutual actions of currents and magnets. The plan has the advantage in this case, not only of making a connexion readily and securely, but also allows the moving part to turn without friction.

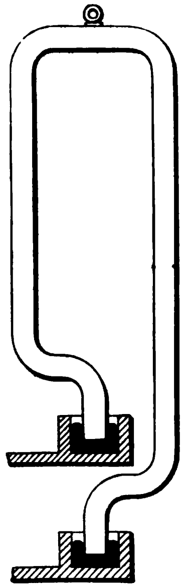


Fig. 15. The moveable frame of a Siemens dynamometer.

38.] Mercury, moreover, lends itself readily to the arrangement of laboratory experiments where a number of changes of connexions have to be made.

By drilling shallow holes in a piece of plank with a centrebit, placing a little mercury in these, and connecting them together as required with pieces of copper wire, excellent switches are obtained of any degree of complexity.

The tendency of mercury to become foul, when exposed to air or kept long in contact with other metals, is well known, and the following extract from Roscoe and Schorlemmer's treatise may be found useful:—

'The mercury of commerce usually contains a certain proportion of dissolved foreign metals, and these impurities give rise to the "tail" seen when the metal is allowed to run over a slightly inclined surface. Impure mercury, when shaken with air, yields a black powder, caused by the oxidation of the metallic impurities, and this film of oxide incloses a small globule of the liquid metal. The surest mode of freeing mercury from these foreign metals is to distil it, the surface being covered with iron filings to prevent the spitting of the metal, which, however, cannot be completely avoided. It is a curious fact that when small quantities of lead or zinc are present in the mercury the rate of distillation is much diminished; other metals do not possess this retarding influence. Mercury may be more easily purified by treating it with dilute nitric acid, the impurities being the first to dissolve. In order to bring the mercury into contact with the acid it is either exposed in thin layers in a shallow vessel, or is frequently shaken up with the acid. The following improved method for the purification of mercury by nitric acid has been suggested by L. Meyer: the metal is allowed to flow in a very thin stream from a small opening in a glass funnel into a wide glass tube, 1·25 m. high and 5 cm. in diameter, which contains a mixture of water and 100 cbc. of nitric acid. A narrow tube is fastened to the bottom of this, from which the pure metal flows; it has then only to be washed with water and dried. The above operations may have to be repeated several times, and the metal, if pure, must leave no residue when dissolved in pure nitric acid, evaporated to dryness and ignited [1].'

Mercury should be kept with its surface covered with dilute sulphuric acid. This has no action, when cold, either upon the mercury or upon any copper connexion-pieces in it. It protects the surface of the mercury from the air,

[1] Roscoe and Schorlemmer, *Treatise on Chemistry*, vol. ii. p. 392.

and tends to dissolve out some of the metallic impurities. Any liquid which has no action on mercury may equally be used to protect it from the air: paraffin oil acts well, but is not so easily removed by washing and drying with blotting-paper as the acid, and has of course no purifying effect.

39.] The property of paraffin oil of reducing the surface tension of mercury, probably explains the curious fact that a glass tube holds more mercury if it is wetted with paraffin oil before the mercury is inserted, and also why the length

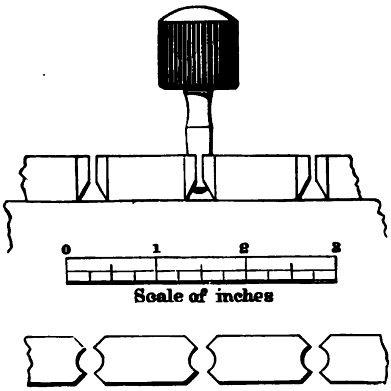


Fig. 16. Taper plug and blocks.

of a jet of mercury to the point where it becomes electrically discontinuous is much greater if the jet is projected into paraffin oil than into air.

40.] Of the various methods of making electrical connexion by pressing two clean metallic surfaces together, the most important is that of inserting a taper plug in a hole between two brass blocks. If the plugs fit the holes well, and are quite clean, the contact is sufficient for all work except the comparison of standards. The ordinary form of this arrangement is shown in the illustration (Fig. 16). Two brass blocks, which are to be placed in electrical connexion or disconnected at will, are attached firmly to an

insulating slab—commonly of ebonite. The space between is occupied by a circular hole of about one-third or one-half of the total width of the blocks. This hole is tapered so that it is smaller at the bottom than at the top, and the edges of the blocks are chamfered, or bevelled away from the sides and from below in the way shown in the drawing. The connexion is made by inserting a brass taper plug into the hole with some pressure, the friction between the brass surfaces preventing the plug being forced out, if the taper be not too coarse. The pressure obtained is fully sufficient to ensure a good contact and the turning motion of the hand in inserting the plug serves to slightly grind together the surfaces of the plug and the blocks. The best results are only obtained from this arrangement when the plug fits the hole very perfectly, and when the surfaces of the plug and the blocks are well polished. If the fit be not a very good one, uncertain and variable contacts will occur, and the plug requires so much pressure to fix it firmly in its place that the blocks are needlessly wrenched. On the other hand, if the surfaces are not well polished, they will be found to wear very quickly in use, when the fit of the plug becomes imperfect.

The practical details of the means by which a good result is secured are worth some attention. In making the plugs it is best to cut the screw first which is to go into the insulating handle, and afterwards to turn the taper end, chucking them on the screw. They should not be fixed in the handles till the metal work is quite finished. The only point of importance in turning these is the method of gauging them. The gauge is constructed of two thin steel plates, separated by two distance pieces or pillars, the plates having holes in them of different sizes to fit the correctly tapered plug. A very small error in the taper of the plug will be detected by its shake when tried in this gauge. Figure 17 shows its form and construction.

The gauge may be conveniently made of such a size that the small end of the taper plug just passes through it, so that it will serve to test the diameter of the plug as well as the angle of its taper.

When the slide-rest of the lathe has been set at such an angle that a taper is turned which fits the gauge, any number of plugs can be turned afterwards without any further setting or adjustment.

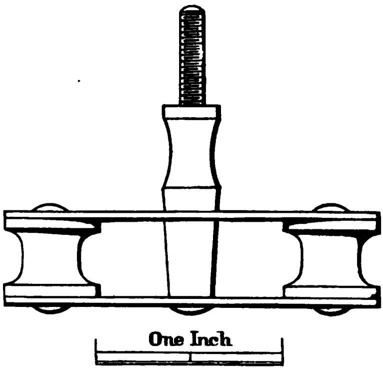


FIG. 17. Gauge for taper plugs.

41.] As to the dimensions of these plugs, a taper of 1 in 10 is very commonly used, meaning by this an increase of 1 inch in diameter for every 10 inches measured along the axis of the plug. For a large size of plug a somewhat finer taper is better, as the thickness of the brass blocks is not generally increased in proportion to their width and to the size of the plugs, and the holes are consequently rather short in comparison with their diameter. These larger plugs are commonly employed for fixed apparatus in constant use, such as large standard dial bridges, or commutators fixed against the wall of a testing-room; the smaller ones for portable apparatus or where a large number of small parts are placed near together.

The sizes used in the Instrument Department of the Silvertown Telegraph Works are the following:—

(*a*) A plug whose taper part is ·6 inch long and of mean diameter ·35 inch with a taper of 1 in 12.

This is the largest size commonly used.

(*b*) A plug whose taper part is ·625 inch long and of mean diameter ·25 inch with a taper of 1 in 10.

This is the size most used, and is a standard size of the Post Office Telegraph Department.

(*c*) A plug whose taper part is ·5 inch long and of maximum diameter ·25 inch with a taper of 1 in 10.

This size differs very little from (*b*), and is not much used.

42.] The ebonite handle of a diamond form commonly used with these plugs is a very bad one. It is inconvenient for constant handling, since its sharp edges make the fingers sore, its large turning leverage is not required, it is not thick enough to secure the handle properly, and it is rather troublesome to make. The form of handle shown in Fig. 16 is more convenient.

43.] Next as to the hole between the blocks into which the plug is to fit. Two points have to be attended to closely. One is the fixing of the blocks, the other the reaming out of the hole. The blocks should be very firmly screwed to their support by large screws, and if that support is of ebonite special precautions must be taken. In the first place, ebonite under strain takes slowly a sub-permanent set; consequently, if the blocks of a resistance box be screwed tightly down, and left for a couple of days, they will be found to require screwing up again. Accordingly, the slab with its blocks should be put on one side for a day or two after they are put together to allow of this second screwing up. Also if the ebonite slab is thin, or drilled with a great many holes, it may become curved under the pressure of the screws and blocks, becoming convex to the side on which the blocks are. This opens the taper of all the plug-holes. The slight warming of the ebonite when the coils receive their final bath of paraffin wax tends to increase this curvature.

It is, however, partly corrected when the slab, with its blocks, is screwed down into its wooden case. It is important, then, that the taper holes should not be finally reamed out until the whole box is finished and screwed together.

44.] The reamers or broaches are of three kinds, called first, second, and finishing broaches. The first broach is used for tapering the holes in the solid brass frame before it is cut up. The second broach is used after the blocks are screwed on their slab and before the coils are mounted. The holes are left by this sufficiently accurate for a final test on the coils when in their places, but are not very smooth. The final broach is used for the last operation on the box. The three forms are shown in Fig. 18.

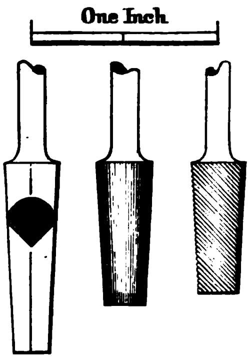


Fig. 18. Reamers or broaches.

The first broach may be made in the form of a smooth cone with two flat sides, somewhat similar to the common five-sided reamers. By cutting the two flats which form the cutting edge at an angle of 90° the remainder of the section is a semicircle. Since this broach is used before the brass is screwed on the slab, it can be thrust fairly through the hole without meeting with any obstruction, and no particular limits of length need be observed. In the case however of the second and finishing broaches, which are used after the blocks are placed on the slab, the dimensions of the thin ends must be carefully observed so that when they have passed clearly into the space cut below the block, and before they touch the ebonite, the hole is just the required size.

The second broach may be conveniently made with straight flutings about as coarse as the milling on the

edge of a half crown. It is equivalent in its action to a fine milling tool. This broach brings the hole nearly to its finished size, but leaves it rather rough.

The finishing broach, which is the most important of the three, is similar to the last, but the flutings are finer and are cut spirally. The spirals are cut as a left-handed screw, so that as the broach is turned into the work, it has no tendency to make spiral grooves in the direction of its own motion, and so to screw itself into the metal. Also at the edges where the saw-cut between the blocks intersects the conical hole, the whole length of one of the flutings of the broach does not come into action at once, and a burr or roughness along the edge is avoided. Both these last principles are well recognized in milling machinery, the first in the arrangement by which the tool turns in an opposite sense to the feed, and the second in the construction of spirally-fluted cutters. The fluting of the finishing broach may be about the same as that of a bastard file, but of course the flutings are not produced by a chisel, but by a V-cutter traversed in a lathe, or on an universal milling machine. The tool requires care in hardening, as its truth of form is important, and it cannot be ground afterwards.

A very convenient appliance for turning these broaches is an engineer's bench brace. The motion of the crank gives just the steady slow continuous motion required, while the slow feed provided in the brace for drilling hard metal is a very suitable one for the work.

45.] For some purposes a better insulation, and one more readily cleaned, is required than is afforded by the small surface of ebonite between two consecutive blocks made on the system just described. If the blocks are mounted on ebonite pillars, it is true they will be well insulated from one another, but it is practically impossible to make these stands so rigid that the blocks will not open under

the pressure of the plug; and if they can open the plug contact is not trustworthy. Two methods of making plug commutators to meet this difficulty are in use. In one of these, called the split plug system, shown in Fig. 19, the two blocks or plates between which contact is to be made are supported by ebonite pillars one above the other. Holes are bored through the plates, and reamed out so that their surfaces form parts of the surface of one cone. Through these can be placed a long taper plug, one or both of the parts of which that correspond with the two plates are split with

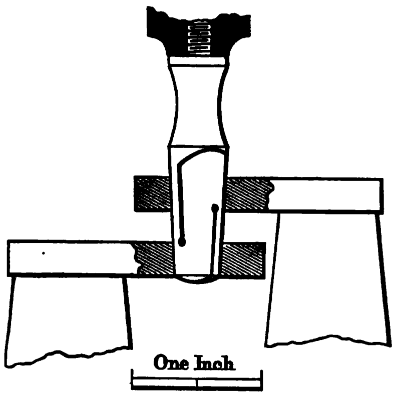


FIG. 19. Split plug and blocks.

a fine saw, and slightly opened. This is to be pressed into its place so that both split parts are compressed. By this means is secured a long, firm contact between each plate and the plug, while if the plugs were solid and not split, it might possibly fit one block very firmly and scarcely touch the other. This plan is commonly used for making what are known as Swiss commutators, which are switches used in telegraph offices for connecting any one of a number of lines 1, 2, 3, &c., to any one of a number of instruments A, B, C, &c. They are made by screwing a number of long bars, one for each instrument, on a board, and placing on supports above, and at right angles to them, in the

manner of a lattice another set, one for each line. Taper holes are drilled at all the points where the upper and lower sets cross, so that by inserting one of the split plugs described above, any bar of the A, B, C set is electrically connected to any one of the 1, 2, 3 set.

46.] The second method of connecting blocks mounted on ebonite pillars is used by Messrs. Elliott, Brothers, and is exceedingly good.

The blocks are mounted on ebonite pillars side by side, with a hole for a taper plug between them, and to prevent them from opening when the taper plug is inserted, a cup ring which is carried loose on the shank of the plug and pressed down with a spiral spring drops over two projections worked on the blocks, forming parts of one circular flange ring. This falls into its place just before the plug is thrust home, and prevents the blocks opening at all when the latter is fully inserted. In fact, this forms even a better arrangement for making a connexion than inserting a plug between two blocks fixed on a solid slab, as the plug fits if anything better, and the additional cup ring is at least as good a connexion as the plug. It is illustrated in Fig. 20.

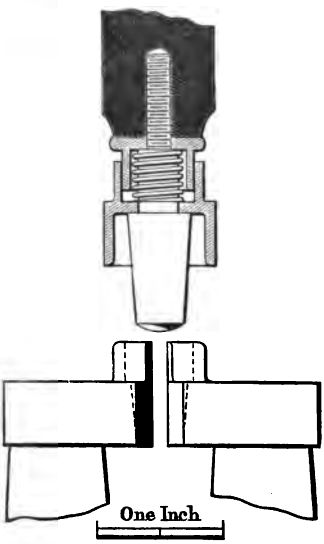


FIG. 20. Capped or ringed plug.

CHAPTER VI.

Wheatstone's Bridge.

47.] The measurement of the electrical resistance of conductors is made in one of several ways, and which of these is to be used is determined primarily by the dimensions of the resistance to be measured; but the method employed

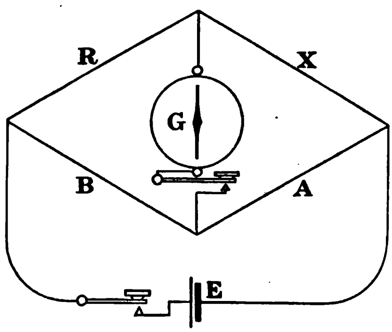


Fig. 21.

almost universally for all resistances which are neither very large nor very small is that known as Wheatstone's bridge, being named after its inventor, the late Sir Charles Wheatstone. The construction and design of apparatus for this purpose is the subject of the chapter. The apparatus has been made in a variety of forms, which will be described by reference to the annexed diagram (Fig. 21). The resistance to be measured—which is marked x—is placed in

a closed circuit with three other known resistances, A, B, and R, of which one or more are adjustable. To two opposite angular points of the quadrilateral thus formed are connected the poles of a battery E, having a contact key in its circuit, while to the two other angular points are connected the terminals of a galvanometer, which also has a key in its circuit, that may be either a contact key or a short circuit key. The method depends on the theorem that when the relative values of A, B X, R are such that no current from the battery flows through the galvanometer, then $\frac{X}{R} = \frac{A}{B}$. The method of use is to adjust the resistances A, B, and R, till on closing first the battery key, and then the galvanometer key, no deflection of the galvanometer needle takes place. The proof of the theorem, with a discussion of the values that should be given to the various resistances for making the measurement to the best advantage, is given in an appendix, and we are now only concerned with the forms of apparatus for realizing the method.

48.] It will be observed that the resistances of neither the battery nor the galvanometer appear in the formula from which the value of X is obtained. Therefore their exact values are not of importance, and, subject to considerations of convenience and expediency, any galvanometer or battery may be used for any given bridge. They may be considered as quite independent parts of the apparatus, and will not be discussed in connexion with the arrangement of the adjustable resistances. This arrangement consists essentially of three circuits, connected in series with one another, containing known resistances, and provided with terminals to which can be connected the battery, the galvanometer, and the conductor whose resistance is to be measured.

49.] A great many forms of the apparatus, more or less convenient, have been proposed, and a considerable number

are commonly used. All these may be divided into two classes: the first consists of those in which A and B remain constant quantities during the process of measurement, while R is adjusted; the second of those in which R remains constant during the measurement while the ratio $\frac{A}{B}$ is the adjustable quantity. In the first class come most of the portable instruments, in the second all those in which a sliding contact piece is used moving over a wire stretched on a divided scale, and also those arrangements in which Kelvin's and Varley's slides form the adjustable ratio.

50.] In designing these arrangements it is essential to provide that no unknown resistances shall be introduced into the quadrilateral circuit XRBA, either when connecting the instruments together, or when making the adjustments; and so the three resistances R, B, A, are very commonly mounted together on one slab, and permanently connected, or at least careful provision is made for connecting them together and to the unknown resistance. In the instruments that come in the first class—viz. those in which A and B are fixed during the operation, and the adjustment made by changing the value of R—a choice of several resistances is usually provided for both A and B; these are all powers of 10, and multiples and submultiples of one another. The quantity $\frac{A}{B}$ is then always a power of 10, and the value of $R\frac{A}{B}$ can be read directly from R by placing the decimal point in the proper place. The value of R can commonly be read to four significant figures.

51.] In Fig. 22 is shown the arrangement of a well-known form of instrument, called the Post Office resistance box, from its being commonly used in the English Postal Telegraph service. Twenty-two resistance coils, whose values in ohms are indicated by the engraved numbers, are

arranged in series with one another, and connected to brass blocks separated by taper holes, in which plugs can be inserted, so that any coil can be short circuited: besides these the series of blocks is broken at two other points where plugs may be inserted, viz. between the blocks *c*, *d*, and at the point marked INF. or infinity. When the instrument

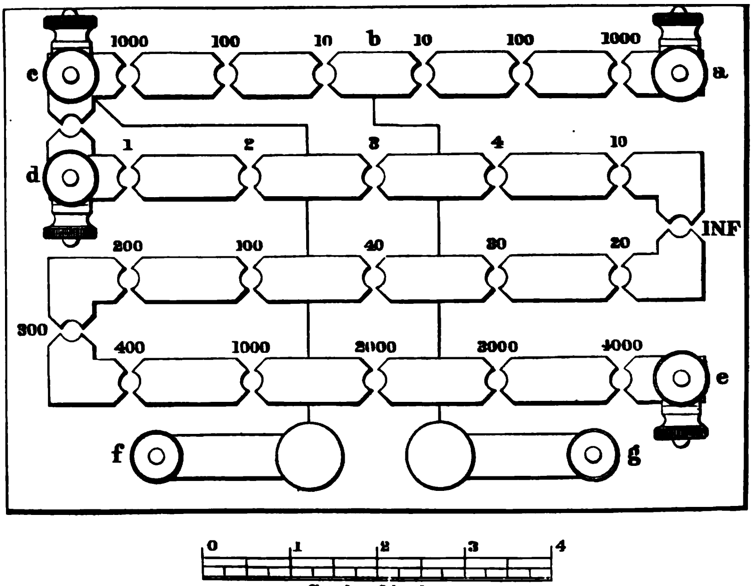


FIG. 22. Post Office resistance box.

is used as a bridge, the former of these plugs is always inserted: the latter allows an infinite resistance to be opened in the circuit between *d* and *e*, which it is occasionally convenient to do. Double terminals are attached to the blocks at *a*, *c*, *d*, and *e*. The resistance to be measured is connected to the terminals *a* and *e*, so that we have a closed circuit *abcdea* consisting of four conductors—viz. the external resistance *ea*, and the resistances *ab*, *bc*, and *de*. These form the quadrilateral of our first diagram (Fig. 21), in which *ab* is represented by A, *bc* by B, *de* by R, and *ea*

by X. The galvanometer and battery are connected to the two pairs of terminals *af* and *eg*. When the balance is obtained the external resistance is equal to $\frac{ab}{bc} \times de$.

By plugging two of the holes between *a* and *b* the resistance *ab*, or A, may be made either 10, 100, or 1000 ohms, and in the same way *bc*, or B, may be given any one of the same three values. Consequently, by proper selection, $\frac{\text{B}}{\text{A}}$ or $\frac{ab}{bc}$ may have any of the values $\frac{1}{100}$, $\frac{1}{10}$, 1, 10 or 100.

Again, the resistance coils between *d* and *e* may be plugged so as to give a resistance of any integral number of ohms between 0 and 11,110. That this can be done is readily seen by dividing the sixteen coils into the following sets; 1, 2, 3, 4,—10, 20, 30, 40,—100, 200, 300, 400,—and 1000, 2000, 3000, 4000. Taking one set, say 100, 200, 300, 400, we can select from them coils whose sum is any integral number of hundreds between 0 and 1000; and similarly with the other sets. Thus in such a resistance as 9347 ohms, the 9000 can be composed of coils selected from the fourth set, the 300 of coils from the third set, the 40 of coils from the second set, and the 7 ohms of coils from the first set.

52.] Sometimes the set is numbered 1, 2, 2, 5, instead of 1, 2, 3, 4, but there is little to choose between the two arrangements: either admits of any integral number from 0 to 10 being obtained from the set, and with the 1, 2, 2, 5 arrangement resistances of 3 ohms and 4 ohms are obtained by plugging two holes and leaving two open, instead of plugging three holes and leaving one open. 5 ohms is obtained by plugging three holes and leaving one open, instead of plugging two holes and leaving two open. One plan can hardly be said to be more convenient than the other. We may remark that these otherwise convenient arrangements, if we desire to obtain the largest resistance possible with a given number of coils, are not economical;

subject of course to the condition that we can select coils from the set whose sums will make every integral number, up to the sum of all the coils in the set. From this point of view the arrangement 1, 2, 4, 8, 16, &c., ascending in powers of 2, is the most economical, sixteen coils of this series giving 65,535 ohms against the 11,110 ohms of the other series, or nearly six times the resistance. For use in a balance the most economical set of weights is the series 1, 3, 9, 27, &c., ascending in powers of 3; but this series is not available for our purpose, as its use depends upon our being able to put some weights in one pan, and others in the other, and thus to use the weights as positive or as negative quantities at will. To obtain, however, the largest possible resistance with a given number of coils is not an object of much importance, and the accepted arrangement is the most practical.

53.] Since then in this case B may have any resistance from 1 ohm to 11,110 ohms, and since $\frac{A}{B}$ may be any power of 10 between $\frac{1}{100}$ and 100, the instrument may apparently be used to measure any resistance from 1 ohm $\times \frac{1}{100}$, i. e. ·01 ohm, up to 11,110 × 100, i. e. 1,111 000 ohms. We say 'apparently,' since in the case of the inferior limit, viz. ·01 ohm, we obtain only one significant figure in the result, while even if one more significant figure is obtained, by observing the swing of the galvanometer needle, the accuracy of the result may be affected by so large a number of plug and screwed contacts occurring in a circuit where so small a resistance is under consideration. And for the superior limit the values of the resistances in the A and B circuits are too low for advantageously taking very high measurements. The practical range of resistance measureable on this instrument is from 100,000 ohms to 1 ohm.

54.] Two keys, or tappers, are placed on the slab, in front of the brass blocks and terminals, either of which, on being

pressed, places the terminal at its end in connexion with the stud in the slab under the ebonite knob or handle. One of the studs is connected to the point *b*, between the A and B circuits, and the other to the terminal *c* between the B and R circuits, as is shown by the engraved lines on the diagram. The terminals *f* and *g* of the two keys are to be connected to the battery and galvanometer, and the terminals *a* and *e* to their other poles. For measurements which are well within the range of sensitiveness of the instruments, it is not material which is the battery key and which the galvanometer, but if the largest deflection of the galvanometer needle obtainable with a given error of adjustment is desired, we are guided by the following rule. Whichever of the two—the battery and galvanometer—has the highest resistance, and this in practice is usually the galvanometer, is to be connected between those angular points of the quadrilateral ABRX, where the two highest resistances and the two lowest respectively meet. Another way of stating this rule is given in Appendix I, page 171.

For measuring resistances

between	1 and	10 ohms	with	a bridge	ratio	10 : 1000
or between	10 and	100	„	„	„	10 : 1000
„	100 and	1000	„	„	„	100 : 1000

the galvanometer and battery should be connected as in the annexed diagram (Fig. 23).

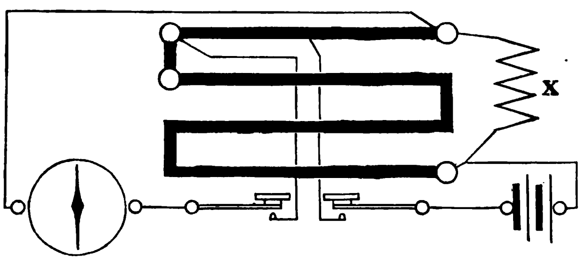


Fig. 23.

For measuring resistances

between 100 and 1000 ohms with a bridge ratio of	10 :	100
or between 1000 and 10,000 „ „ „	10 :	10
or of	100 :	100
or of	1000 :	1000
„ „ 10,000 and 100,000 ohms with a bridge ratio of	1000 :	100
or of	100 :	10
or over 100,000 ohms with a bridge ratio of	1000 :	10

the galvanometer and battery should be connected as in the annexed diagram (Fig. 24).

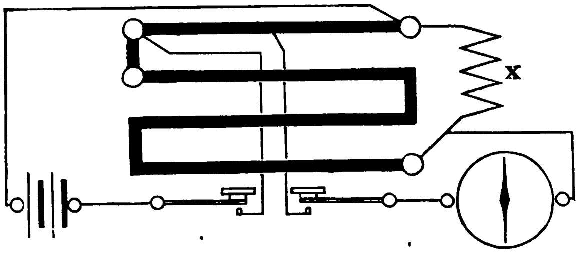


FIG. 24.

The two circuits A, B, which form two adjacent sides of the quadrilateral are collectively termed 'the bridge'; and the ratio $\frac{A}{B}$ by which R is multiplied to obtain the value of X is called 'the bridge ratio.'

55.] In three of the above classes of measurements we have a choice as to which bridge resistances are to be employed.

Thus the ratio

10 may be obtained either as	100 : 10, or 1000 : 100,	
1 „ „ „	10 : 10, or 100 : 100, or 1000 : 1000	
·1 „ „ „	10 : 100, or 100 : 1000.	

In this choice we are guided by the following rule: the resistance of the circuit B should be the geometric mean between the resistances of the battery and the galvanometer, and for preference should be higher than this figure rather than lower. The resistance of the galvanometer used with these instruments is commonly about 800 or 1000 ohms, while the battery resistance may be anything from 1 to

20 ohms, and would generally be nearer the lower limit. The figures indicate that the left-hand bridge resistance that represented by B or by *bc* in the diagrams, should in every case be 100 ohms in preference to 10 or 1000 ohms; so for the ratio 10 we should use 1000 : 100, for the ratio 1 we should use 100 : 100, and for ·1 we should use 10 : 100; for the other bridge ratios, viz. 100 and ·01, we have of course no choice.

56.] The Wheatstone's bridge of the Post Office pattern, just described, is portable and compact, with a capacity for measuring a great range of resistances. How large this range is will be realized, if we consider how few measuring instruments there are which can readily deal with two quantities, one a hundred thousand times as great as the other, and the whole of the intermediate values; and will deal with considerable accuracy with quantities one-tenth of the minimum and ten times as great as the maximum. It has, however, the disadvantage that the R circuit, which is the one to be adjusted, has a very large number of plugs. Care is required to see that all are tight in their places, and small changes of adjustment are apt to require the movement of an inconveniently large number of plugs: e.g. ten movements have to be made to change from 4000 ohms to 3999.

57.] A mode of construction which meets this difficulty is that known as the dial form of bridge. In this form each set of four coils whose resistances were of relative values 1, 2, 3, 4, and amounted in all to ten, is replaced by nine or ten coils all of the same value. Nine of these coils are connected in series with one another to ten blocks arranged in a ring, with a larger circular block in the middle, and any one of the ten can be connected by a plug to the central one. Thus the resistance between the first block of the series of ten and the central block, may be changed to any value from 0 to 9 by moving the one plug used to make the

connexion; and this first block and the centre are to be regarded as the terminals of the dial. It is convenient for the purpose of occasionally testing the coils to connect a tenth coil between the last block of the ten and the centre, so that, when the plug is not inserted at all, the resistance between the terminals is 10. The annexed illustration (Fig. 25) shows a Wheatstone's bridge, of which the R circuit marked *de* is made on this plan. This circuit consists of five sets of ten coils whose values are 1000, 100, 10, 1. and ·1 ohm respectively. Each set is arranged in the circular form just described, which is commonly called a dial. In the tens dial is shown, diagrammatically, the connexions of the ten coils of 10 ohms each: and the connexions between the several dials are also indicated. The numbers engraved on

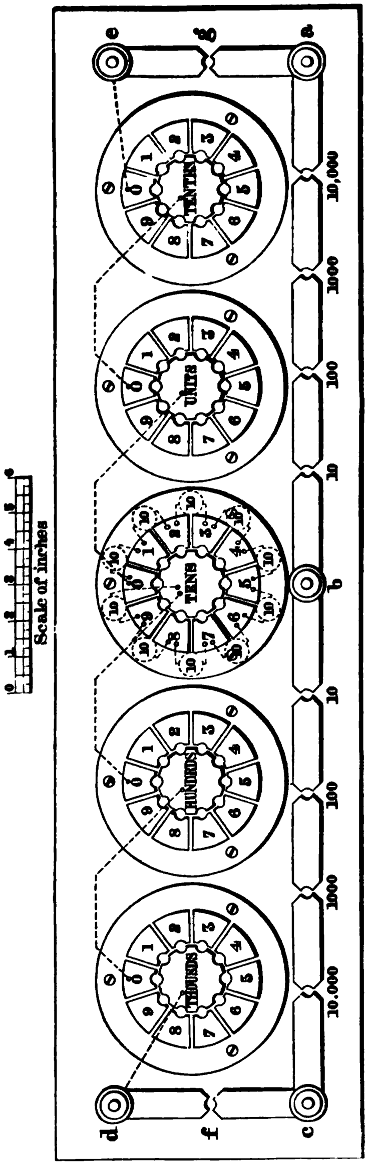


Fig. 25. Dial pattern of Wheatstone's bridge.

the blocks of the outer ring show the resistance of the

dial when the plug is inserted in the corresponding hole, and the resistance of the circuit is read to five significant figures by taking in order the five figures on the dials against the plugged holes. The binding screws *d* and *e* form the terminals of this circuit. The A and B circuits constituting the bridge are placed below the dials, and the ends separated from the terminals of the R circuit by plug holes at *f* and *g*. The resistance to be measured is connected either between the terminals *a* and *e* or the terminals *c* and *d*; and a plug inserted at *f* in one case, or at *g* in the other. The only difference between these two ways of connecting the instrument is, that in one case *ab* is the A circuit, and *bc* the B circuit, so that $\frac{ab}{bc}$ is the bridge ratio; while in the other *bc* is the A circuit, and *ab* the B circuit, and $\frac{bc}{ab}$ is the bridge ratio.

A choice of four resistances, viz. 10, 100, 1000, and 10,000 ohms, is usually given in both arms of the bridge, so that bridge ratios of 1000 and ·001 are available in addition to those in the Post Office box. The bridge coils of 10,000 ohms are, however, added rather with a view to measuring very high resistances than to obtaining the extra bridge ratios.

58.] Where these dials are only fitted with nine coils each, it is true that any resistance can be measured with five significant figures, but two advantages in using the box are gained by fitting the tenth coil. In the first place the plugs can be moved without altogether breaking the circuit, and this is of great importance in dealing with conductors which possess either inductance or electrostatic capacity, or include a variable electromotive force. It may be seen that when only nine coils are fitted there is no circuit through the dial unless a plug is inserted in one hole, while with ten coils the circuit is always continuous. This might of course be done by a permanent connexion

between the block marked 9 and the centre, or even the 9 block omitted and the end of the ninth coil connected to the centre, in which case the resistance would be 9 when the plug was altogether removed; but such a plan would lose the second advantage. This can be best described by an example. Suppose in the course of measuring a resistance we have the dials plugged to give 3290·0, and the galvanometer shows that this is to be increased. Removing the plug from the 'tens' dial, that dial gives 100 ohms, and the whole R circuit becomes 3300·0, and the movement of the galvanometer shows whether this is too high or too low. If only nine coils are fitted in the 'tens' dial, two plugs have to be changed to obtain 3300 ohms, viz. that in the 'tens' to zero, and that in the hundreds to 3; and both have to be changed back again if 3300 proves too much. Thus, to try 3300 ohms four changes are required, instead of lifting one plug and replacing it.

59.] The tenth coil is moreover very convenient for the inter-comparison of the coils in the box, both at the time of and after its construction. For each dial contains a set of ten resistances of the same value, arranged in series with one another, so that each separate coil should be equal to the whole resistance of the next dial on the right, and the whole dial should equal the resistance of each separate coil in the dial next on the left. If a separate test be taken of every coil in the dials, and of each dial, we obtain in all fifty-five measurements.

Of which	10	are resistances of		·1 ohm
	11	„	„	1 ohm
	11	„	„	10 ohms
	11	„	„	100 ohms
	11	„	„	1000 ohms
	1	is a resistance of		10,000 ohms,

and besides these we have the measurements of 8 coils in the bridge.

The agreement with one another of all coils of nominally the same value, and especially the identity of the sum of

the coils in one dial with the average value of those in the next dial on the left, is a very precise measure of how far the coils are multiples and submultiples of one another. It is of course no test of the agreement between any coil and the standard it is supposed to represent; but, for the examination of the bridge coils which are required to be multiples of one another, and whose absolute values are unimportant, the method is perfect. If the coils in the dials are found to agree well in being multiples of one another, the determination of the value of one coil only in terms of the standard suffices to give us all the data of the box. The author prefers, however, to measure one coil at each end of the dials, and uses for that purpose two standards of 1000 ohms and 1 ohm respectively.

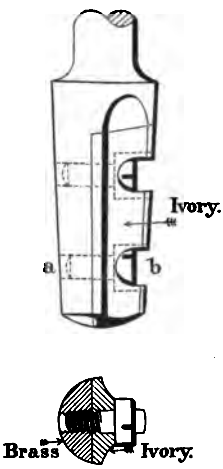
60.] This inter-comparison of the coils depends only on comparing sets of ten or eleven resistances of nominally the same value, and requires no exact apparatus for carrying it out. For the amount by which two resistances of nearly the same value differ from one another is very easily measured, and can be satisfactorily determined with apparatus quite inadequate to give with any accuracy the total value of either, since the difference is not required to be known within the same percentage error as the whole coil would be. E.g. an error of 1 per cent. in the value of a coil of 100 ohms would be serious, while an error of 1 per cent. in the measurement of the difference between two coils of 100·00 ohms and 100·02 ohms, made for the purpose of comparing them, is inappreciable—always supposing that the difference does not appear as an independent factor in our result. To take an analogous case, a set of weights, too inaccurate to weigh 20 lbs. correctly within 1 oz., might yet be used to determine within a quarter of an ounce the difference between two masses of about 20 lbs. each.

61.] Again, in constructing a set of coils on this principle, it is often possible to compensate for errors in the several

coils by a proper arrangement of the order in which they are placed. To take an extreme case, consider the following set of ten coils, each of which is intended to be 100 ohms:

Coil No. 1	100·01	
" " 2	99·97	199·98
" " 3	100·05	300·03
" " 4	99·93	399·96
" " 5	100·09	500·05
" " 6	99·89	599·94
" " 7	100·13	700 07
" " 8	99·85	799·92
" " 9	100·17	900·09
" " 10	99·81	999·90

The right-hand column of figures is the resistance of the dial with the corresponding hole plugged. It is seen that a set of coils, all of which are in error by amounts varying from 1 to 19 parts in 10,000 may be arranged so that there is no error in the dial of more than 1 part in 10,000.



Section a.b.

Fig. 26. Travelling plug for testing dial boxes.

62.] It remains to show how the separate coils in the dials can be measured apart from the others. It is done by the use of a pair of travelling plugs, one of which is shown in Fig. 26, of which only part is of brass, the remainder being ivory or some other hard insulator. The brass part is a binding screw, to which a leading wire is attached. Such a plug can be inserted in a taper plug-hole, only making contact with the metal block on one side, and the resistance between two such plugs placed in consecutive holes is that of the coil between the two blocks. Before using such plugs as these it is necessary to make sure by an experiment that the brass part of the plug is really insulated by the ivory

from the other block, and this insulation is improved by the ivory being shaped as shown in the Figure.

63.] A bridge of this form will generally be only used in laboratories or testing rooms with a good reflecting galvanometer; and this is so amply sensitive that it is not necessary to use the bridge coils that will give the largest deflection for any given error of adjustment if any advantage is gained by using others. As was pointed out in the discussion of the Post Office Box, in article 32 a. the heating of a coil under a given E. M. F. is inversely proportional to its resistance: consequently less error will be introduced into the

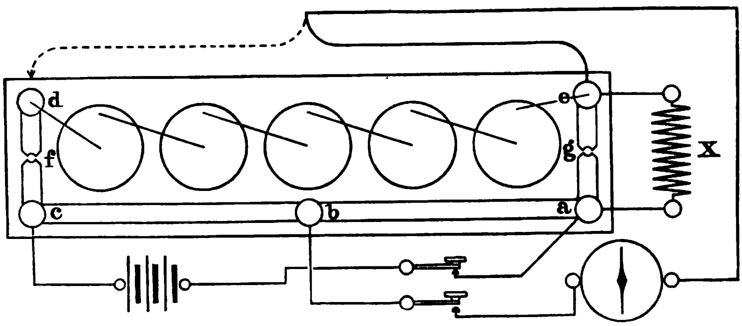


Fig. 27.

result, through the alteration of the bridge resistances by heating, if with the same testing battery we use high bridge resistances in preference to low ones. The 1000 ohm coil may be used as the permanent resistance of the B circuit, and the battery always connected to the two extremities of the bridge: the considerations which lead to changes in the relative positions of battery and galvanometer being neglected.

64.] No keys are usually fitted to an instrument of this kind which only provides the A, B, and R circuits. The connexions of the galvanometer and battery are shown in the annexed diagram (Fig. 27), with their independent keys. The external resistance to be measured, marked X, is

connected between *a* and *e*, and the hole *f* is closed with a plug. If x were connected between *c* and *d*, the *f* plug would be transferred to *g*, and the galvanometer connexion at *e* transferred to *d*, the other leads remaining unchanged.

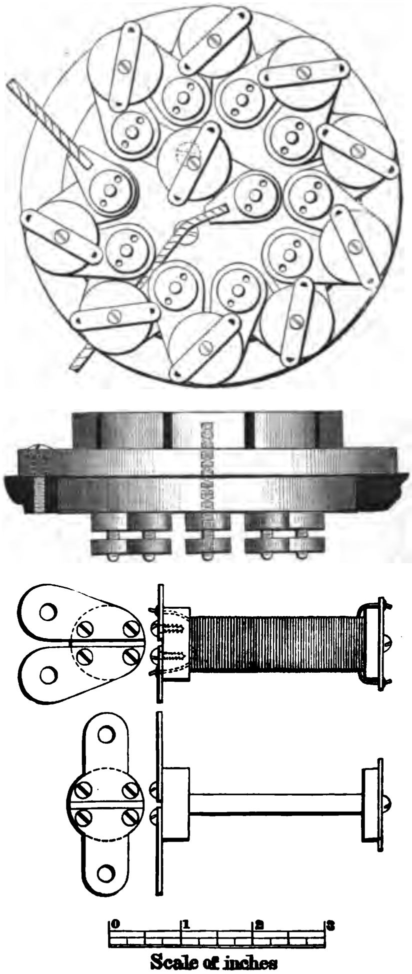


FIG. 28. Details of a dial, and its coils.

65.] The annexed illustration (Fig. 28) shows the construction of a dial with its coils, which are made and

connected on the lines proposed in chapter III, article 26. The form of coil with straight brass plates is used for the bridge coils, as well as for the tenth coil in the dials. The connexions between the dials are of No. 16 copper wire, insulated with silk ribbon, and soldered to stout brass plates.

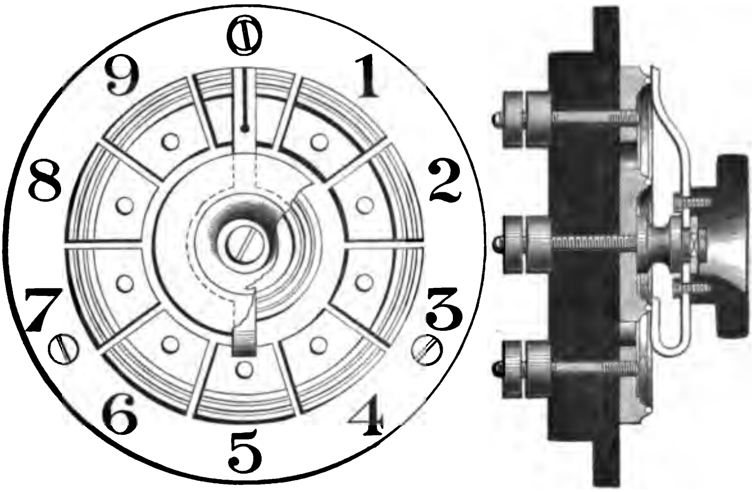


FIG. 29. Arrangement of a dial with sliding commutator.

Fig. 29 shows an arrangement in which a sliding switch is substituted for the plug commutator. This is very convenient, and, if the switch is well made, is practically quite as good as the plugs. It will be seen that the pivot on which the handle turns is not used as a connexion at all, all the connexions being made by the sliding spring. The round wires soldered to the blocks are gold, and the rubbing-piece of steel or of platinum.

CHAPTER VII.

The Slide Wire, or Metre Bridge.

66.] The next class of apparatus for comparing electrical resistances is that form of Wheatstone's bridge which is adjusted by altering the ratio of the two arms. This includes in the first place a large number of instruments, in which a stretched wire forms the whole or part of the bridge circuit, connexion with which is made through a sliding contact; and in the second place a valuable instrument, designed by Lord Kelvin and Mr. Varley, used for many purposes besides the one we are concerned with. Omitting reference for the present to Kelvin and Varley's slides, we confine our attention to instruments which are adjusted by sliding a contact along a stretched wire. Generally these instruments consist only of the bridge circuit, with a convenient arrangement for adjustment of its two arms, and means of connecting the conductors which are to form the X and R circuits of the diagram of Figure 21. They may be looked upon as designed only for comparing two resistances, viz. the two that form the X and R circuits, and not for determining the absolute value of either. Of course if one of the resistances is known in terms of the standard unit, this instrument gives the measure of the other, but no standard coils form an essential part of the design. They are analogous to ordinary balances, which are machines for comparing two masses,

as contrasted with a spring balance or steelyard which give in pounds the mass of a body weighed. The construction of almost all standards of physical quantities depends on the continued copying, and aggregation of copies, of a single unit, and requires some means of closely comparing such copies with one another and the original. The arrangements we are describing are peculiarly well adapted for making the comparison between two resistances of nearly the same value, and are almost universally employed for the work.

67.] For the purpose of description we shall take first the metre bridge, which presents in a simple form the points of construction and manipulation common to them all. The metre bridge shown in Figs. 30, 32 consists of a board about 4 feet long, on which are laid five copper or brass plates numbered 1, 2, 3, 4, 5. Erected on 1 and 5, and soldered to them, are stout pillars between which is stretched a wire of uniform section. A wooden straight-edge, engraved with a scale, is fixed underneath at such a height that the wire lies upon it. The wire may be of exactly the same length as the divided scale, in which case the instrument lends itself—somewhat indifferently—to a more extended application than is to be recommended. When used for comparing nearly equal coils to the best advantage the exact length of the wire is immaterial. The scale shown in the diagram is 40 inches long, divided into 400 equal parts numbered from 0 to 200 in each direction from the middle. The length is more commonly 1 metre divided into millimetres—and from this the instrument received its name of the metre bridge—but the author prefers divisions of ·1 inch to be subdivided by estimation. A sliding frame, shown in detail in Fig. 31, can be moved along the straight-edge and carries four attachments: (i) a tapping key with an ebonite handle, used to make contact with the wire; (ii) a terminal for connecting a lead; (iii) an adjustable pointer

for reading the scale; (iv) a pinion engaging in a rack attached to, and running the whole length of, the scale,

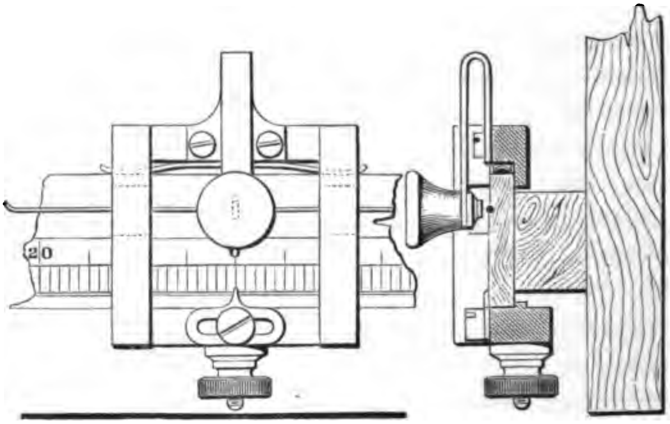


Fig. 31. Sliding contact frame of the metre bridge.

which is turned by an ebonite milled head, and so moves the whole slider.

Between plates 1–2, and 4–5, are placed resistance coils of as nearly as possible the same value, shown here as 100 ohms: and the coils whose resistances are to be compared are connected between plates 2–3, and 3–4. The

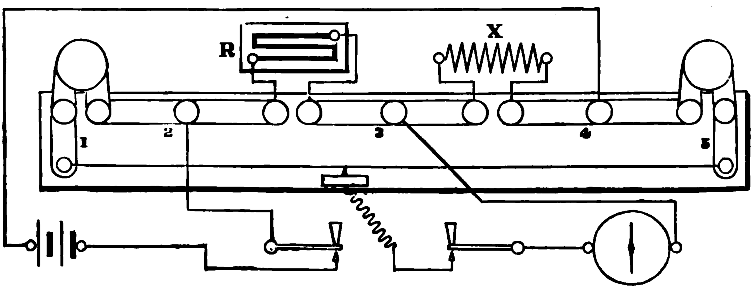


Fig. 32. Connexions of a metre bridge.

battery with its key is connected between plates 2 and 4, and the galvanometer between plate 3 and the tapping key on the slider. The connexions are shown on the annexed diagram (Fig. 32). Referring again to the diagram (Fig. 21)

the coil between plates 1 and 2, together with that part of the wire which lies between plate 1 and the slider, constitutes the A circuit; the coil between plates 4 and 5, with the wire between plate 5 and the slider, is the B circuit; whilst the coils between the pairs of plates 2–3, and 3–4, are respectively the X and R circuits. When the slider stands at such a point that no deflection of the galvanometer needle is observed on making contact on the wire with the tapping key, the resistances are such that R : X :: A : B; or, in other words, the ratio between the resistances, placed in the gaps 2–3 and 3–4, is equal to that in which the resistance between the plates 1 and 5—i. e. the total bridge resistance—is divided by the tapping key. The ratio of A to B is adjustable within such limits as are allowed by the length of wire stretched upon the scale. Consequently if we can determine from the figures on the engraved scale the proportion in which the bridge is divided by the slider, we simultaneously obtain the ratio of the resistances in the gaps 2–3, 3–4.

68.] The method of using the apparatus is to connect the two resistances to be compared in the gaps between the plates 2–3 and 3–4, and to move the slider until a position is found in which no deflection of the galvanometer is obtained on tapping the key. The bridge ratio, and the ratio of the two coils, are then obtained from the reading of the pointer.

69.] The details of the construction and use of the instrument will be taken in the following order:—

(i) The method of determining the bridge ratio from the scale reading (Articles **70, 71**).

(ii) The determination of the middle point of the bridge (Articles **72–74**).

(iii) The methods of connecting the coils to the bridge and of interchanging them in double balancing (Articles **75, 76**).

(iv) The material for, and the calibration of, the bridge wire (Articles **77–81**).

(v) The elimination of the thermoelectric effects produced at the junctions of the bridge (Article **82**).

70.] ***The determination of the bridge ratio from the scale reading.*** We will first take a very simple case. Let us suppose the coils at the end of the scale wire to be replaced by copper straps, so that the stretched wire itself forms the whole of the bridge, and let the length of the wire coincide with the length of the scale. This case is shown diagrammatically in Fig. 33. Then the part of the wire to the left

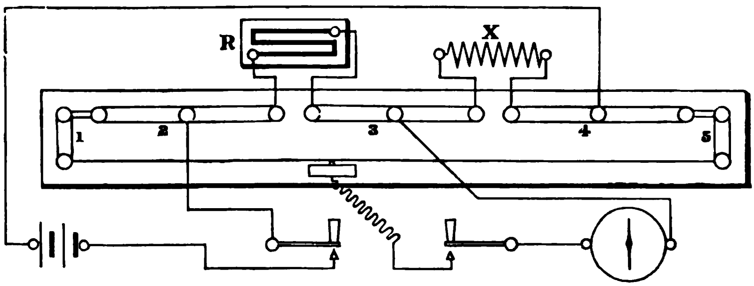


Fig. 33. Metre bridge without bridge coils.

of the slider is the A circuit, and that to the right the B circuit. Suppose the slider to have been adjusted to give no galvanometer deflection, and that the index points to n divisions to the right of the centre or zero, the bridge ratio $\frac{A}{B} = \frac{200+n}{200-n}$. This expression when n is small $=$ $1 + \frac{n}{100}$ approximately. Hence since $\frac{X}{R} = \frac{A}{B}$, if X is nearly equal to R so that n is small, a certain number of divisions to the right of zero, when balance is made, indicates that X exceeds R by that number per cent., while a small number of divisions to the left indicates that X falls short of R by that number per cent. This rule it must be remembered is approximate only when n is small. If the

deviation is great, the full formula given above, viz. $\frac{200+n}{200-n}$, must be employed to obtain the bridge ratio.

This method it is seen only compares the two resistances R and X with another to 1 per cent., or to such a submultiple of that amount as may be obtained by an estimation reading between the scale divisions. Such a subdivision is not advisable, as in addition to the inequalities of the wire—always to be found—the soldering of the scale wire near its ends may render its resistance not strictly proportional to the scale divisions throughout its length, and an attempt to reach great accuracy by fine subdivision of the scale may prove futile.

For such work, however, as the examination of a long series of hanks of telegraph wire, all supposed to be of the same gauge, or other similar operations of a commercial character, a degree of accuracy of 1 per cent. will often be sufficient.

71.] Next let us consider the case when we have the resistance coils shown in Fig. 32 in the gaps 1–2 and 4–5. Then the bridge circuit consists of the two coils and the scale wire. Suppose the total resistance of the bridge thus composed to be $2Q$, so that Q is the resistance of one coil and half the scale wire: and suppose the resistance of the scale wire between two consecutive divisions of the scale to be kQ, so that k is a small quantity. Then, if the balance be found when the slider is n divisions to the right of the zero or centre, the bridge ratio is $\frac{Q+nkQ}{Q-nkQ}$, which $= 1+2nk$ approximately, unless n is very large. Now, if we compare two resistances of nearly the same value in this way, the number of divisions n to the right or left shows at once by how many times a quantity which is $2k$ times the value of either, the one exceeds or falls short of the other.

As an example of this, consider the following arrangement

for the comparison of the coils of resistance boxes. The scale 40 inches long is divided into 400 parts; the scale wire which is of platinoid of 18 mils. diameter measures a little over 2 ohms for the 40 inches, and the bridge coils are each $49\frac{1}{2}$ times the value of the bridge wire. For the purpose of this discussion we will take the wire to be 2 ohms, and each bridge coil to be 99 ohms, so that the whole bridge resistance is 200 ohms, and each half is 100 ohms; then the resistance between two consecutive scale divisions is ·005 ohm, so that $2k = \cdot 0001$.

Now suppose two coils in the gaps 2–3 and 3–4 are compared, and a balance obtained with the slider four divisions to the right of the centre. The bridge ratio is

$$\frac{100 + 4 \times \cdot 005}{100 - 4 \times \cdot 005} = 1 + 4 \times \cdot 0001 \text{ very nearly}$$
$$= 1 + 4 \text{ ten-thousandths.}$$

So any given number of divisions to the right or left when balance is obtained shows that the coil in the gap 2–3 exceeds or falls short of the coil in gap 3–4 by that number of parts in 10,000. In comparing resistance coils with the standard in the course of manufacture, small multiples of one ten-thousandth part are precisely the kind of quantities to be determined, and so a scale which allows these to be read off directly is of great service in such work. The space of one-tenth of an inch between the divisions may readily be subdivided by estimation to at least one-fifth part, which gives the relative values of the two coils under comparison within 2 parts in 100,000. But if such small differences are to be observed, it would be better to use a thicker wire on the scale, whose resistance would be a smaller fraction of that of either of the bridge coils. Thus, if the coils were each 99·9 ohms, and the wire—of platinoid—were of about 59 mils. diameter, measuring ·2 ohms for the 40 inches, every division of one-tenth of an inch on the scale would indicate a variation of 1 part in

100,000. The range of the instrument, however, would thereby be reduced from the comparison of coils which agree within 2 per cent. to those which agree within ·2 per cent.

72.] *The determination of the middle point of the bridge.* This does not necessarily coincide with the middle point of the bridge wire, or the zero of the scale, since the two bridge coils will not generally be of exactly the same resistance, nor the scale wire absolutely uniform.

The point is determined from the consideration that the ratio of any two resistances, obtained when they are placed in gaps 2–3 and 3–4, must be the same as that obtained when they are interchanged; or, in other words, that the reading of the pointer when balance is obtained must be the same on one side of the zero, as it is on the other side after the interchange of the coils. The method is as follows. Having fixed the moveable pointer on the slider in some position taken at random, put any two coils of nearly the same value in the gaps 2–3 and 3–4: next adjust the slider till there is no galvanometer deflection, and take the reading of the pointer. Suppose it is *a* divisions to the right. Interchange the coils in the gaps 2–3 and 3–4 and adjust again. Let the reading now be *b* divisions to the left. Then the contact piece would touch the middle point of the bridge, if placed half-way between the two positions, i.e. when the pointer is opposite to a reading on the scale half-way between *a* divisions to the right and *b* divisions to the left. Having placed the slider in such a position that the pointer gives this reading, and that consequently the contact key touches the middle point of the bridge, loosen the set screw that fixes the pointer, and reset it to point to the zero of the scale. It is now fixed in such a way that, when the pointer is at zero, the contact exactly bisects the bridge, and we shall obtain the same reading to the right of the zero in one arrangement of the two coils as we obtain to the

left of the zero in the other. To take an example: two coils were inserted in the gaps, and the slider adjusted. A reading was obtained $14\frac{1}{2}$ divisions to the right. The coils were then interchanged and the slider again adjusted. The reading was then 12 divisions to the left. Placing the slider in the intermediate position between the two, i.e. with the pointer indicating $\frac{3}{4}$ division to the right, the set screw was loosened and the pointer moved to the zero, in which position the contact key exactly bisects the bridge circuit. With the two coils placed in the first position, the balance would be obtained with a reading $13\frac{3}{4}$ divisions to the right, and in the second position with a reading $13\frac{3}{4}$ divisions to the left.

73.] In very accurate work this process of double balancing is carried out with every pair of coils compared, and the ratio of their two values is taken to be the mean of the results obtained. Of course, if this is to be always done, the pointer need not be adjustable, since the mean of the two readings will give the correct ratio of the values of the coils, however great may be the error of either reading. This method of double balancing is analogous to the method of double weighing with an ordinary balance.

74.] Another method of double balancing is much more economical in point of time than the last when several nearly equal resistances have to be compared to a standard coil. This consists of first balancing the standard coil against any convenient resistance, and afterwards replacing the standard successively by each of the coils to be compared with it. Thus if on the bridge described above, the standard coil, balanced against the resistance first taken, gives a reading d divisions to the right, and the others which are to be compared with it d_1, d_2, d_3, &c., we know that they exceed the standard by (d_1-d), (d_2-d), (d_3-d), &c. parts in 10,000 respectively, without taking into consideration the value of the coil against which they have

all been balanced, or the exact position of the scale zero. This method only involves one adjustment more than the number of coils to be compared, while the method first described involves twice as many. It is analogous, of course, to the method of weighing by the use of a counterpoise.

75.] *Methods of connecting the coils to the bridge, and of interchanging them in double balancing.* The illustration shows only ordinary screwed terminals. These are, however, well adapted for making firm connexion, as the screw is stout —it is drawn of the size No. o B.A. standard thread—while

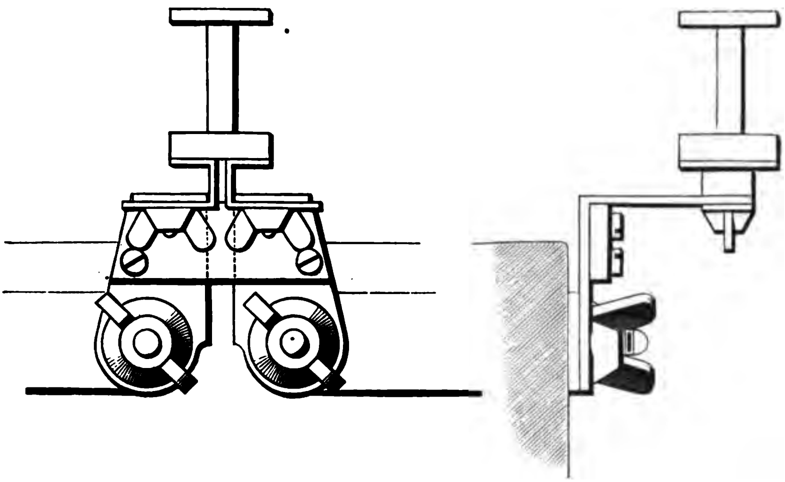


Fig. 34. Method of attaching coils to the terminals.

the nut has a large clamping surface, and is of a form easily screwed up tightly by the fingers. It has already been explained that this method of attachment of coils is, with care, quite sufficient for any purpose short of the highest accuracy. For the connexion of such coils as are illustrated in Fig. 7 an attachment of the kind shown in Fig. 34 may be used. It is drawn with a coil in position for testing. The angle piece to which the coil is attached may be set

at the distance required by the holes in the terminal plates by means of a slot at A. Where great accuracy is required mercury cups are always used instead of the terminal screws. These are of copper firmly soldered to the plate of the bridge, and the coils are connected by means of copper terminal rods, plunged into the mercury in the cups and held down by clamps, in the way described in article 36. These cups are shown in the illustration to the next article.

76.] Several arrangements have been used for changing the connexions of the apparatus in the process of double balancing to save handling the coils. One very convenient form, designed by Mr. Latimer Clark, is shown in Fig. 35.

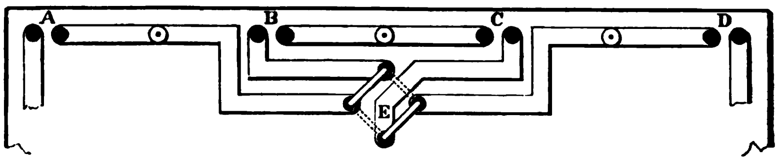


FIG. 35. **Arrangement of mercury cups for reversing the connexions.**

The bridge coils are connected to the mercury cups at A and D, and the coils to be compared to those at B and C. The four mercury cups at E are connected in pairs by two copper loops. By moving these loops to the situations shown in dotted lines, the positions of the two coils under comparison are reversed with reference to the bridge, without handling them at all.

77.] *The material for, and calibration of, the bridge wire.* The material employed by the committee of the British Association was platinum iridium. Its advantages are :

(i) Great hardness, so that it is not liable to be worn by the pressure of the contact key.

(ii) That it is not liable to be oxydized by the air.

(iii) That it is not readily acted on by mercury.

Its expense is the only objection to it, but this is not a considerable item in the cost of a complete set of instruments for testing resistance. For most purposes platinoid or German silver is quite suitable, and its wear under the contact key is not very quick.

78.] The method designed by Professor Carey Foster for calibrating the wire consists in obtaining a series of points such that the resistance between each consecutive pair is the same. It is carried out by the simultaneous calibration of two wires. Two metre bridges lettered in Fig. 36 A_1, A_2,

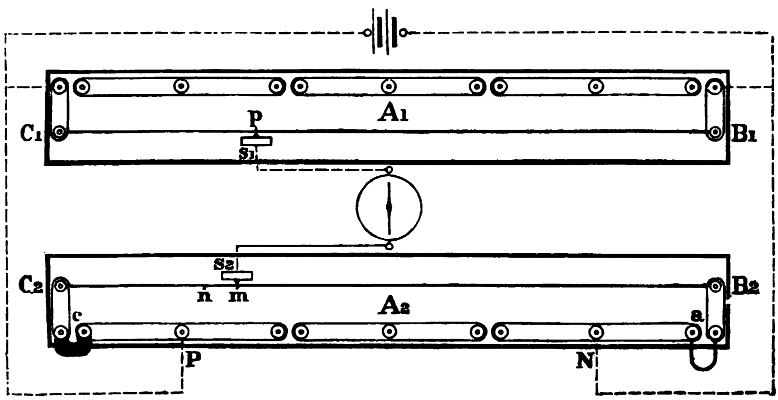


Fig. 36. Carey Foster's method of calibrating a scale wire.

are connected to a galvanometer and battery. The galvanometer is connected between the sliders s_1, s_2, and the battery passes parallel currents through the two scale wires. In A_1 all the gaps are left open, and the battery is joined to the ends of the wire. In A_2 the two inner gaps are open, while for connecting *a* and *c* two pieces are provided; one of these—called the connector—shown at *c*, is a copper strap having very little resistance; the other—called the gauge—shown at *a*, has a small resistance. In A_1 the resistance between the battery terminals consists of that of the scale wire; in A_2 it consists of the scale wire with the resistances

of the gauge and connector added at the ends. The total resistance in A_2 is always the same, but its arrangement depends on which end is occupied by the gauge, and which by the connector. The procedure is as follows:

(i) The connector being at a and the gauge at c, place s_2 at B_2, and adjust s_1 till there is no galvanometer deflection. This of course shows that s_1 and s_2 are then at the same potential. The positions of s_1 and s_2 are noted.

(ii) Putting the gauge at a and the connector at c, keep s_2 fixed, and move s_1 towards c_1 till there is again no deflection. Note the position of s_1.

(iii) The gauge and connector are again interchanged; s_1 is kept fixed and s_2 is moved towards c_2 till there is again no deflection. The position of s_2 is noted.

Operations (ii) and (iii) are alternately repeated until s_1 and s_2 have been moved the whole length of their scales.

Then the resistances between the points noted on either wire are equal, but those on one wire are not equal to those on the other. The galvanometer in these operations is only used to test the equality of the potentials of the two sliders, and might be replaced by an electrometer, if that were equally sensitive. The theory of the method is as follows. Let s_1 be at p, as shown in the diagram, s_2 at m, and the gauge at a: suppose operation (ii) to have been just completed so that the potentials at p and m are equal. The interchange of the connector and the gauge is the first part of the next operation, and the effect of this is to reduce the resistance between the battery pole N and s_2 by an amount equal to the difference of the resistances of the gauge and connector, and to increase the resistance between s_2 and P by the same amount. This changes the potential at s_2, so

that it is no longer equal to that at p, and its movement to a new position n, where the potential is the same as that of p, is the second part of operation (iii). Since then the potential at m, when the gauge is at a and the connector at c, is the same as that at n when the gauge is at c and the connector at a, it follows from Ohm's law that the resistance mn is equal to the difference of the resistances of the gauge and connector, i. e. to a fixed constant quantity.

To put the argument in symbols, let the resistance of the gauge be g and of the connector c; let the resistance of B_2m be r, of mn be ρ, and of B_2C_2 be R. Let the potential of P be E and of N be zero. Then with the gauge at a and the connector at c, the resistance of the whole circuit NB_2C_2P is $g+R+c$, and of the part NB_2m is $g+r$. The potential at m is then $E\dfrac{g+r}{g+R+c}$. With the gauge at c and the connector at a, the resistance of the whole circuit NB_2C_2P is unchanged, while that of the part NB_2n is $c+r+\rho$. The potential at n is consequently $E\dfrac{c+r+\rho}{g+R+c}$. Since these two potentials are both equal to that at p, we have

$$g+r = c+r+\rho, \text{ i. e. } \rho = g-c$$

a constant quantity.

79.] This process then divides the whole wire B_2C_2 by a series of points, the resistances between which are all equal to one another and to $g-c$: while B_1C_1 is divided by a series of points whose potentials, when connected as above, are equal to the potentials of the points on B_2C_2. Since the differences of potential at various points along a conductor conveying a current are proportional to the resistances between them, it follows that the differences of potential between the successive points noted on B_2C_2 are all equal, and consequently the resistances between the points noted on B_1C_1. Hence each wire is divided into a series of sections of equal resistance, though the sections of the two

wires are not necessarily equal. If the battery can be depended upon to maintain a perfectly constant E. M. F. and resistance during the test—e. g. a good secondary battery—it would be convenient to determine a series of points along each wire, which are of the same potential, without interchanging the connector and gauge; and afterwards, making the interchange, determine another series of points on the B_2C_2 wire which are severally distant from those of the first series by the same resistance. This would give a number of sections of the wire of the same resistance, which would not be coterminous, and may overlap one another. The exact values of the connector and gauge are not important: their difference should be such as to make the sections of the scale wire a convenient size, say one-fortieth of the whole wire, and they should be so small as not to seriously reduce the available range of potential on the scale wire.

80.] Another method of calibrating a wire by the use of two sliders, which is less laborious than this in operation, and does not require the use of two wires to be calibrated together, will be found in chapter IX, article 101.

81.] In good wires the difference in length between any two sections is always small, and may be very small indeed. For metre bridges used in the way described here, where the scale wire itself is short, and forms only a small portion of the whole bridge, the examination should be made; but if a serious want of uniformity is found. it would generally be better to replace it with a properly drawn wire than to construct and use a table of corrections. For the very long wires, however, used in potentiometers, calibration is always required.

In the account published by M. I.-René Benoit of the construction of the standards of resistance for the French postal service, may be found a very instructive description of the calibration and gauging of the glass tubes

employed, which offers many analogies with the gauging of wires[1].

82.] *The elimination of the thermo-electric effects produced at the junctions.* These junctions are very numerous; the scale wire with its supports and with the slider, the first being of platinum iridium or platinoid, the latter of brass or copper; the copper cups and bars with the mercury; the wires in the resistance coils with their terminals and the solder. If the whole apparatus is at identically the same temperature, these electromotive forces balance one another, and have no resultant effect: but the passage of the testing current, and the warmth of the hand, will be quite sufficient to produce appreciable thermo-currents, which become serious when the adjustment of the balance is nearly complete. In the particular case when the whole apparatus is at one temperature, except the slider which has become warm under the hand, the only effective thermo-electromotive force will be at the sliding contact, and will act in the circuit connecting the slider to the central bar, a circuit usually occupied by the galvanometer. In Appendix III. will be found a discussion, due to Mr. R. T. Glazebrook, of the circumstances in which this becomes important, and of a method for determining its value. But if the circuit between the slider and the central bar 3 be occupied by the testing battery, and the galvanometer be connected between bars 2 and 4, their usual places being reversed, this small electromotive force will act only as an addition to the battery, and will not at all disturb the conditions of the test. This method is suggested, but not recommended, by Messrs. Stewart and Gee in their book on *Elementary Practical Physics.* It has this disadvantage —which they do not consider important—that the whole testing current is passed through the very small point of

[1] *Construction des Étalons Prototypes de résistance électrique,* par M. I.-René Benoit. Gauthier Villars, Paris, 1885.

contact of the slider, and might conceivably injure the scale wire at that place: and further, takes no account of electromotive forces due to differences of temperature in other parts of the apparatus, differences which it is not safe to neglect.

To eliminate these effects they propose two methods of testing, both executed with the galvanometer connected in the usual way to the slider.

i. To obtain a balance twice in succession, reversing the testing battery between the operations, and to take the mean of the two readings as the one required. In the one case the thermo-current will aid, while in the other it will oppose, the battery current, and hence the mean of the two will give a correct result.

ii. To adjust the slider so that the same deflection is observed when the slider key is depressed, when the battery circuit is closed and when it is not. That deflection is due to the thermo-current, and the slider is then so placed that no part of the battery current passes through the galvanometer circuit. This last method they especially recommend.

Method i. may be objected to on the ground that it requires two operations, which, taken with the method of double balancing recommended by the same authors, involves four operations for each comparison. Method ii. is not a null method.

The method of balancing against a counter-resistance, described in article 74, meets all the difficulties. In this the standard coil is first balanced against some convenient resistance, and then replaced with the coil to be compared with it. Any differences of temperature in the junctions will have the same effect in both operations, provided that the whole mass of each coil is at the same temperature, and will have no effect on the ratio of the values of the two coils deduced from these operations. The junction whose tem-

perature is most likely to change during the operations is that of the slider, and the effect of any current arising from this may be eliminated in the way already described by connecting the battery in the slider circuit instead of the galvanometer. The long ebonite handles provided in the form of slider, illustrated in Fig. 31, both for depressing the key and moving the slider, are designed to keep the hand at as great a distance as possible from the wire.

CHAPTER VIII.

Lord Kelvin's and Varley's Slides.

83.] An instrument was described in the last chapter whose principal features are a stretched wire and a sliding contact. A current of electricity from a battery being passed through the wire, the different points present a series of potentials of every value intermediate to those of the extremities: and that of any other object is obtained by determining with an electrometer, or galvanometer, a point on the wire which has the same potential. This method of determination depends for its precision on the length of wire which corresponds to any given difference of potential, and hence for a given difference of potential at its extremities the wire should be as long as possible.

A longer wire offers the further advantage over a short one of the same weight, that through its greater resistance less current is used to obtain the same fall of potential along its course, and the heating of the wire is less. E. g. Let a platinoid wire of 40 inches in length, of 59 mils. diameter, and having a resistance of ·2 ohms be replaced by another of the same weight, and twice the length, having one-half the sectional area, and a diameter of 42 mils. The resistance of the second wire is four times that of the first. Hence with the same difference of potential at its ends, and offering the same range of potential for comparison with other objects, only one fourth of the current is required, twice the length of wire corresponds to any given potential

difference, and the heat developed in the wire per square inch of its superficial area, and consequently its rise of temperature, is reduced in the proportion of about 6 : 1.

The following rules may be found useful in practice for the currents which may be carried by bare, and freely exposed, cylindrical wires of platinoid.

The rise of temperature is proportional to the square of the current.

The diameter d in mils. of a wire to carry a current of c ampères with a rise of temperature of 5° Ft. is given by the equation $d^3 = 500^2 c^2$.

According to these rules a bare platinoid wire of 63 mils. will carry a current of 1 ampere with a rise of temperature of 5° Ft., and carry 2 amperes with a rise of 20° Ft.: and an electromotive force of ·16 volt may be applied to the ends of a metre bridge wire of 63 mils. diameter, and 40 inches long, with a rise of temperature of 5° Ft. Very long wires stretched on cylinders were employed by Sir Charles Wheatstone for his rheostats, and potentiometers are made by arranging long calibrated wires on frames in such a way that contact can be made with them at any point, and a great range of continuously graduated potential obtained. This construction is somewhat inconvenient, since it requires a large frame, and also involves the calibration of a great length of wire, which is, moreover, exposed throughout its whole length and liable to injury.

84.] The long wire may be replaced by a series of coils, mounted in one box, and connected up to a series of terminals or contacts, connexion with which may be made by a switch. Contact with the circuit can, of course, only be made at the terminals between each pair of consecutive coils; but a special arrangement, to be described, enables the contact to be made in a way which virtually subdivides each coil into any desired number of parts. This apparatus is known as Kelvin's and Varley's slides.

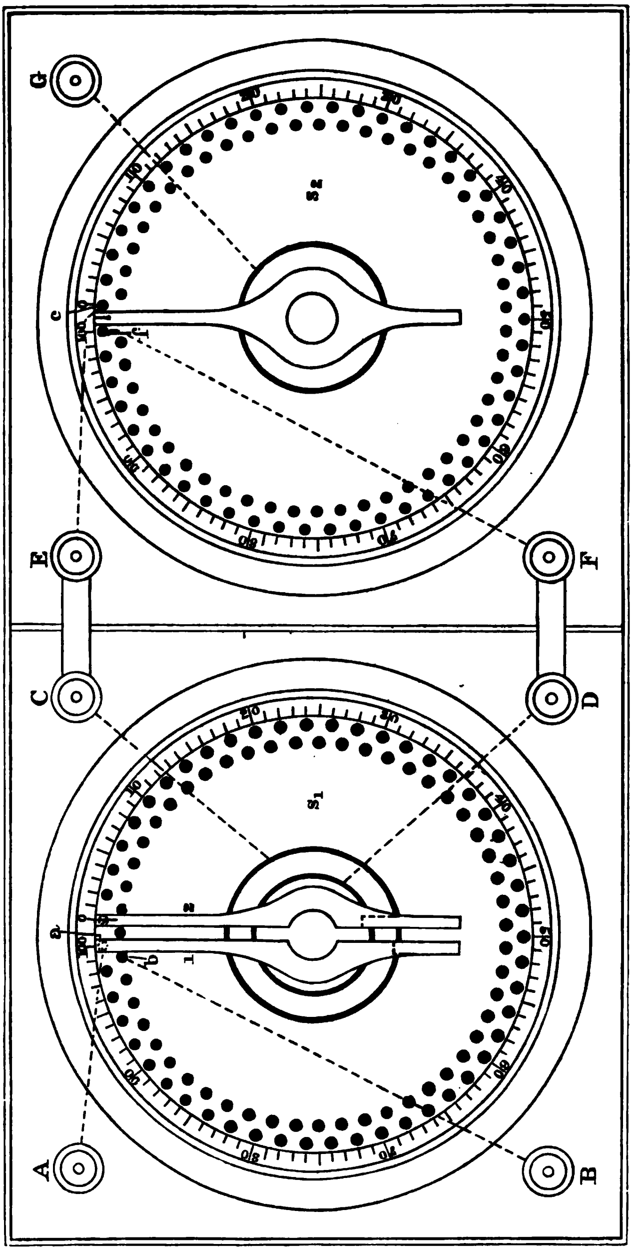


FIG. 37. Kelvin's and Varley's slides.

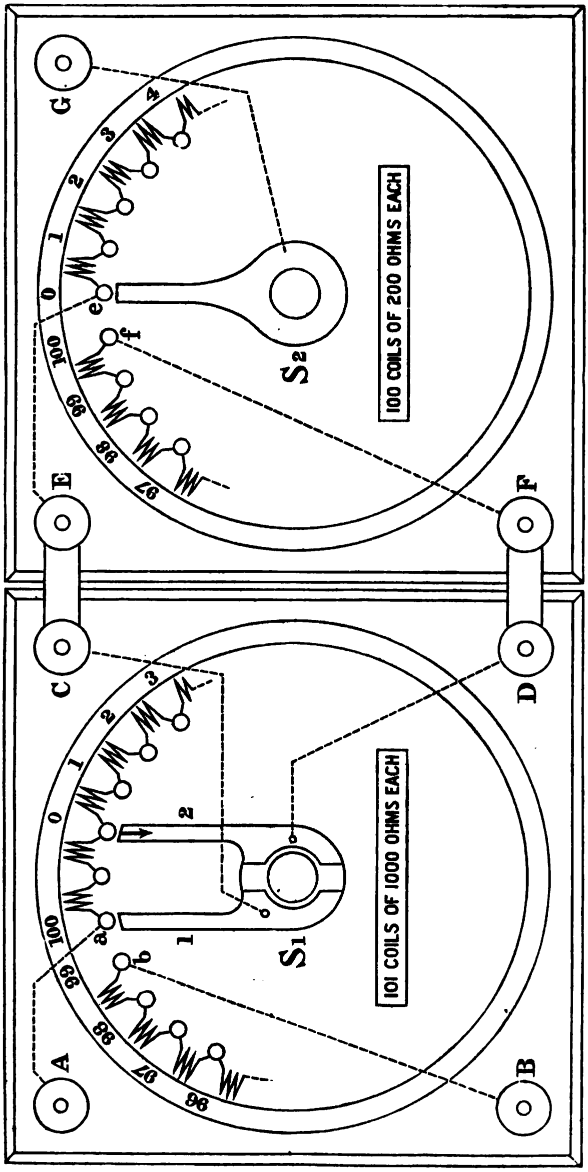
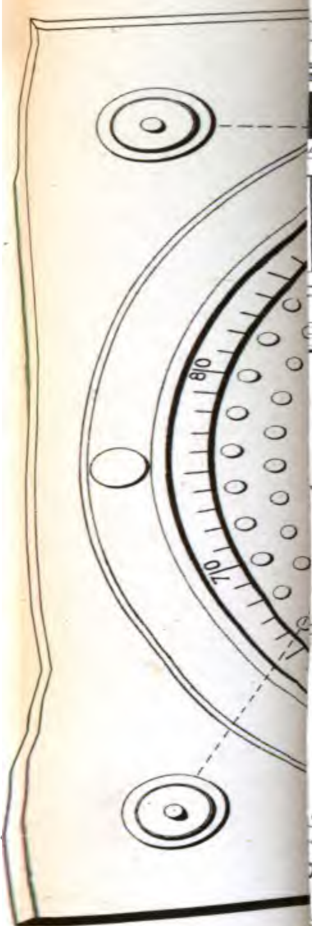


FIG. 378. Kelvin's and Varley's slides. In this view the positions of the contact studs are displaced to allow the connexions of the coils to be seen.

The slides which are illustrated on Figs. 38, 39, 40, and shown diagrammatically in Figs. 37, 37 a, are in two parts. The case on the left marked S_1 contains a set of 101 coils, each of 1000 ohms resistance, connected in series, to give a total resistance of 101,000 ohms, with a contact piece at the junction of each consecutive pair of coils. One end of the series is at the contact *a*, and is connected to terminal A: the other end is at contact *b*, and is connected to terminal B. Over these 102 contacts, arranged in a circle from *a* to *b*, sweep a pair of switch springs attached to one centre piece, but insulated from it, and from one another. The two contacts which these touch in any position are not consecutive, but intercept another between them, so that the resistance between the two springs is always 2000 ohms. The two springs are connected to the terminals C, D.

The case on the right marked S_2 contains a set of 100 coils, each of 20 ohms resistance, connected in series to give a total resistance of 2000 ohms, with a contact piece at the junction of each consecutive pair of coils. One end of the series is at the contact *e*, and is connected to the terminal E; the other end at the contact *f* is connected to the terminal F. Over these 101 contacts, arranged in a circle from *e* to *f*, sweeps a single switch connected to terminal G.

85.] The two cases are placed side by side, and the terminals C–E, and D–F, are connected by stout straps. By this arrangement the whole resistance of the box S_2, which is 2000 ohms, is placed in parallel circuit with that between the switch springs of S_1, which is also 2000 ohms. The total resistance between the springs is thus reduced to 1000 ohms, and the total resistance between the terminals A, B to 100,000 ohms. In this way the switch springs of S_1 and the connected terminals intercept one-hundredth part of the total circuit between A and B, and can be made to intercept it in any part of the circuit. Consequently if a battery be connected to A and B so that a current passes through the



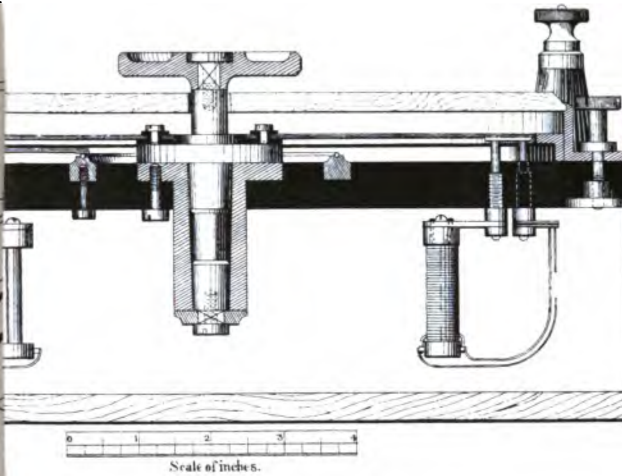
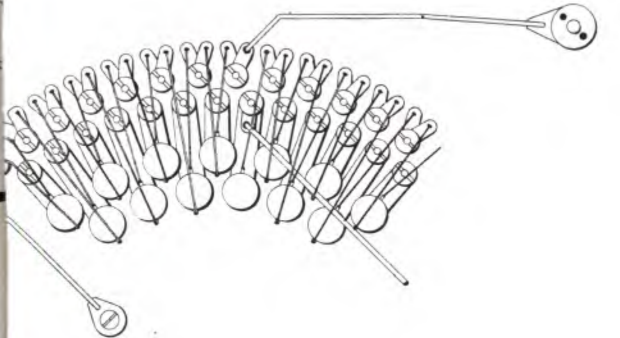


FIG. 38

view of one of Kelvin and Varley's slide boxes



I

FIG. 40

s and arrangement of some of the coils of FIG. 39

whole circuit, the difference of potential between any two consecutive contact studs in s_1 will be equal to that between the switch springs, and to one-hundredth part of the total difference between A, B. The difference of potential again between E, F, i. e. between the switch springs, is subdivided into 100 equal parts by the 101 contact studs in s_2; so that the difference of potential between two consecutive studs in s_2 is one-hundredth part of that between E and F, and one ten-thousandth part of that between A and B. By the adjustment then of the switches in s_1, s_2, the switch spring of s_2, and its terminal G, can be placed at any one of the points which divide the fall of potential between A and B into 10,000 equal parts.

86.] As an example, suppose the terminals of a cell with an electromotive force of 1 volt connected to A and B, and suppose the potential of A to be 1, and of B to be zero. Let the spring 1 be placed at the sixteenth contact stud counted from A. Then the potential of spring 1 is ·16, and of spring 2 is ·17, the difference of potential between them, and between E and F, being ·01 volt. Let the switch of s_2 stand at the forty-seventh contact stud numbered from C. Then the difference of potential between E and the switch spring is ·0047 volt, and the difference of potential between the switch spring, or its terminal G, and the terminal A is ·1647. In other words, if the fall of potential from A to B be divided into 10,000 equal parts, the two numbers indicated by the switch in the scale of s_1, followed by those indicated on the scale of s_2, give the number of those parts between A and G; subtracting this number from 10,000 gives, of course, the potential between G and B.

87.] Figs. 38, 39, 40 show the mechanical construction of the apparatus, the two parts being similar except in a few points. Taking Fig. 38, which is a sectional view of s_2, all the parts are seen to be mounted on an ebonite slab. In the centre is placed a socket or bearing, in which the axis

turns carrying the switch spring. The axis is let into the socket with a taper fitting to prevent its becoming loose with wear. Round the centre is placed a brass ring, to which is soldered a gold wire, and this ring is permanently connected to the terminal G. Round this again at some distance are arranged alternately in two circles the 101 contact studs, tipped with gold. Connexion is made between these studs and the brass ring by means of a long contact spring, carried by, but insulated from, the central axis. This spring is tipped with platinum at the points where it rubs on either the studs or the ring. Two different unoxidizable metals, such as gold and platinum, will make good contact with one another, and slide over one another with but little friction, while two surfaces of platinum tend to stick to one another and wear very fast.

The contact studs, which are screwed tightly into the ebonite from below the slab, consist of the cylindrical part carrying the gold tip, of the screwed part in the ebonite, and of a screwed shank fitted with a nut. The bobbins are of boxwood, and are carried by stout brass plates, which are alternately long and short, on the screwed shanks of the inner circle of studs. Each bobbin carries two wires, the inside ends being soldered to the brass plates, and the outside ends to two studs fixed in opposite sides of the lower cheek. To these studs are also soldered stout copper wires, which are connected to brass end plates, or lugs, screwed tightly under the outer circle of nuts. Each wire has a resistance of 20 ohms, so that each bobbin carries 40 ohms, and a resistance of 20 ohms is connected between each stud of either row and the studs of the other row on each side of it. The slab forms the top of a wooden box which serves as its support, and protects the coils from injury. The switch and contacts are covered with a glazed cover, to the inside of which is attached a ring on which the numbers indicating the coils are engraved.

Fig. 39 shows a plan of s_1, which is similar to s_2, except in the following points:—

(i) There are two switch springs, making contact with two brass rings, one outside the other, which are connected to separate terminals.

(ii) There are 101 coils of 1000 ohms each, and 102 contact studs.

Fig. 40 shows the arrangement and connexion of some of the coils of s_1 seen from below.

88.] Owing to the similarity in construction and circumstances of all the coils of either set, it is possible to adjust them with great accuracy, since, as was pointed out in Article 60, a number of copies of one particular resistance can be obtained much more easily than a set of coils which are exact multiples of one another. Moreover, the fact of their all being wound on bobbins of the same size, with similar wire, and at all times carrying the same current, reduces the probability of any differences between them developing with lapse of time. The only point to which particular attention is required is the equality between the whole resistance of s_2 and that of any two coils of s_1. This may be secured by adjusting the coils of s_2 first, and afterwards adjusting the coils of s_1 to agree with them.

The method of winding two wires on each bobbin avoids the necessity of double winding each separate wire.

The apparatus is of great value, and would probably be much more widely used were it less costly. It is used to replace the bridge in measuring very high resistances, e.g. in comparing the insulation resistance of a cable with a standard megohm. It is used also for comparing the capacities of condensers.

CHAPTER IX.

COMPARISON OF LOW RESISTANCES.

89] THE ratio to one another of two very low resistances is determined on the same principle as of higher resistances, viz. by observing the differences of potential at pairs of points on the conductors when the same current of electricity is passed through both. We have, by the definition of resistance, that the ratios of the resistances between the several pairs of points is the same as that of the differences of potential.

Ohm's law permits us to neglect any consideration of the dimensions of the current.

In Wheatstone's bridge the conductors under comparison are connected more or less perfectly to terminal screws, mercury cups, or other appliances; and the difference of potential observed in measuring the resistance of the conductor is that between two of these terminals, or cups. Now the form of the conducting circuit, at the point where the wire under examination is connected to the terminal, is of a complicated shape, being composed partly of the terminal and partly of the wire, and includes, moreover, except where mercury cups or soldered joints are used, connexions made by clamped surfaces. The exact value of the resistance introduced at the terminal is small, but, on account of the complicated form of the part, difficult to estimate, whilst that of clamped surfaces cannot be

ascertained. There is no difficulty in assigning a major limit to the values of such resistances, which will be negligible in comparison with the resistance of the wire under measurement, so long as that has not less than a certain value. When, however, the resistance to be measured is small, the unknown resistances introduced at the terminals become important and cannot be neglected.

90.] Lord Kelvin devised the following method of dealing with this problem. The arrangement is shown diagrammatically in Fig. 41.

Suppose we desire to compare the resistances per unit of length of two stout wires AB, CD, one of which is known.

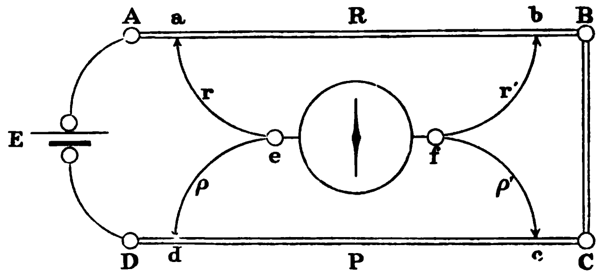


FIG. 41. Connexions of Wheatstone's bridge with auxiliary conductors.

These are connected in series with one another, with a battery E, and a convenient loop-piece BC. The terminals e, f of a galvanometer are connected by what are termed auxiliary conductors to clips or contact pieces a, b, c, d, attached to the conductors at points not very close to the irregular ends A, B, C, D. This last condition ensures that the equipotential surfaces at a, b, c, d are perpendicular to the directions of the wires, so that the potential at the surface, where the clip touches the wire, is that of the cross-section at the point. If R, P be the resistances of AB, CD; r, ρ of ae, de; r', ρ' of bf, cf; and if $r : \rho :: r' : \rho'$; then we have $R : P :: r : \rho :: r' : \rho'$. The demonstration of this theorem is given in Appendix II, with a discussion of the best values

to be given to the resistances of the auxiliary conductors r, r', ρ, ρ' and of the galvanometer.

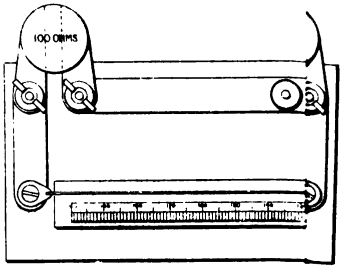
91.] In applying this method, suppose AB to be the unknown, CD to be the known conductor. a, b would be clips or contacts applied to AB at any points desired, provided that they are so far from the irregular ends A, B, that the current is there evenly distributed throughout the section. c, d are moveable or sliding contacts applied to CD, subject to the same condition. r, r', ρ, ρ' are all of such dimensions that any small resistances at the points of contact a, b, c, d, may be neglected, and the ratio $r : \rho$ is equal to the ratio $r' : \rho'$ as nearly as possible. It is shown in Appendix II that, if the resistance bBCc be very low, a small error in the ratio $r' : \rho'$ is negligible. Consequently, if the resistance of the connexion BC be very small, and if b, c be placed near to B, C, respectively, we may neglect any small departure from the exact proportion $r' : \rho'$, provided it be nearly equal to the ratio $r : \rho$, and write $R : P :: r : \rho$.

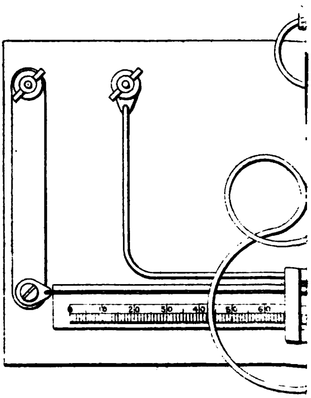
92.] Several authors have described the application of this method to the comparison of very short lengths of conductors [1].

The following description is of a portable apparatus, designed for measuring the resistances of electric light cables wound on drums, or similar conductors, ranging from a maximum resistance of ·36 ohm down to ·001 ohm. The method is convenient for such work, both because the small resistances in question are easily dealt with, and because it does not require the conductors under examination to be brought close to the apparatus, which, with large drums of cable, or armatures of dynamos, may be inconvenient.

The arrangement is shown diagrammatically in Fig. 42 connected to a drum of cable, and a plan of the apparatus is given in Fig 43, which will be found between pages 88 and 89.

[1] Clerk Maxwell, *Electricity and Magnetism*, chap. xi. A. Gray, *Absolute Measurements in Electricity and Magnetism*, vol. i. chap. vi.


100 OHMS


20
30
40
50
60

In this apparatus a scale wire is stretched between pillars A_1, A_2 over a box-wood scale 40 inches long, divided into 400 divisions, in a similar way to the wire of the metre bridge shown in Fig. 30. Two sliding frames S_1, S_2 similar to that shown in Fig. 31 are used, the springs of which are always in contact with the wire. Of the four terminals B_1, B_2, C_1, C_2, the outer ones B_1, C_2 are connected to the ends A_1, A_2 of the scale wire, while the inner ones B_2, C_1 are connected together by a stout conductor. In the centre

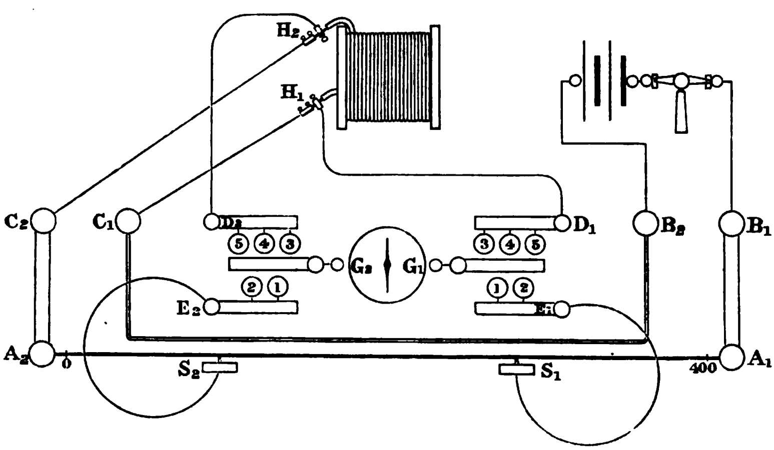


FIG. 42. Connexions of apparatus for measuring low resistances.

of the board is placed a galvanometer, whose terminals G_1, G_2 are connected to the central bars of two similar resistance commutators, placed one on each side. Each central bar can be connected by means of plugs through certain resistances marked in Fig. 42. 1, 2, 3, 4, 5, which are the same for both commutators, to the other bars bearing terminals D, E, which carry leading or auxiliary wires. The leads from E_1, E_2 are connected to the sliders S_1, S_2, those from D_1, D_2 to the clips H_1, H_2. Then GDH, GES are termed the auxiliary conductors, and when the proper

holes are plugged the resistances of the two leads GDH are equal to one another, and also the two GES.

In using the instrument the testing battery and key are connected to B_1, B_2, and the ends of the cable to C_1, C_2: the clips H_1, H_2 at the ends of the leads from D_1, D_2 are attached to the cable conductor inside the points where current from the battery enters and leaves.

Then, referring to Fig. 41, the resistance of the cable between the points H_1, H_2 is the resistance R, that of the scale wire between the sliders P, the resistances of $G_1D_1H_1$, $G_2D_2H_2$ are r and r', those of $G_1E_1S_1$, $G_2E_2S_2$ ρ and ρ', r in this case being equal to r', and ρ to ρ'. Then, when a current is passed from the battery through the system, and S_1, S_2 adjusted till the galvanometer shows no deflection, the resistance of the cable between H_1, H_2 bears the same ratio to that of the scale wire between S_1, S_2, that the resistance of $G_1D_1H_1$ does to that of $G_1E_1S_1$. The two latter being known, and also the values of the scale divisions of the wire, we obtain the value of the resistance of the cable between H_1, H_2.

The provision of commutators at G_1, G_2, whereby the ratio of the resistances $G_1D_1H_1$ and $G_1E_1S_1$, i.e. the ratio $r : \rho$, may be given several different values, allows a greater range of resistances to be dealt with than would otherwise be possible.

93.] The scale wire is of platinoid of 100 mils. diameter, having a resistance of about ·07 ohm between the points 0 and 400 on the scale. We assume it to be exactly ·07 ohm for purposes of description.

The resistance between the galvanometer and either slider is 7 ohms when the plugs are in the holes marked '$\frac{1}{3}$ or 1,' and 2·333 ohms when the plugs are in the holes marked '3 or 9.'

The resistance between the galvanometer and either clip is 12 ohms when the plugs are in the holes marked '9,' it

is 4 ohms when they are in the holes marked '3 or 1,' and 1·333 when the holes marked '$\frac{1}{3}$' are plugged.

Hence, when all the holes—

$$\begin{array}{llll} \text{marked } 9 \text{ are plugged, the ratio} & \frac{r}{\rho} & \text{is} & 9\times\frac{4}{7}, \\ \quad ,, \quad 3 \quad ,, \quad ,, \quad ,, & \frac{r}{\rho} & \text{is} & 3\times\frac{4}{7}, \\ \quad ,, \quad 1 \quad ,, \quad ,, \quad ,, & \frac{r}{\rho} & \text{is} & \frac{4}{7}, \\ \quad ,, \quad \frac{1}{3} \quad ,, \quad ,, \quad ,, & \frac{r}{\rho} & \text{is} & \frac{1}{3}\times\frac{4}{7}. \end{array}$$

The resistance of the whole 400 divisions of the scale wire being ·07 ohm, it is seen that the external resistance required to balance it is $9\times{\cdot}04$, $3\times{\cdot}04$, ·04, or $\frac{1}{3}\times{\cdot}04$, according as the holes marked 9, 3, 1, or $\frac{1}{3}$ are plugged; and the resistance required to balance n divisions of the scale wire is $n\times{\cdot}0009$, $n\times{\cdot}0003$, $n\times{\cdot}0001$, $n\times{\cdot}0000\dot{3}$ in the four cases.

In short, when balance is obtained, the resistance measured is found by reading each scale wire division between the sliders as one ten-thousandth of an ohm multiplied by the number marked against the plugged holes.

94.] It is advantageous to have the greatest length of scale available for any given measurement, and with this arrangement it is seen that resistances can be measured from ·36 ohm down to ·0033 without using less than 10 inches of the scale, i. e. 100 divisions of the scale wire:

·36 ohm down to ·12 is measured with the holes marked 9 plugged.

·12 ohm down to ·04 is measured with the holes marked 3 plugged.

·04 ohm down to ·01333 with the holes marked 1 plugged; and resistance below ·01333 with holes marked $\frac{1}{3}$ plugged, ·00333 being balanced with the sliders 100 divisions apart.

95.] Several points connected with the construction and use of the instrument require special notice:

i. The four flexible leads, called auxiliary conductors or potential leads, connected in pairs to the clips and the sliders, may be conveniently made of a heavy but flexible copper strand, of say $\frac{1}{8}''$ diameter, and covered by an india rubber tube. This is flexible enough to allow the connexions to the cable to be readily made, and the sliders easily manipulated, but is too strong to be easily broken or torn away from its connexions; this is a point of importance, since the resistances of these leads are an essential part of the adjustment of the instrument, and, if they are destroyed, time and care are required to replace them. Their resistance is so low, compared with that of the resistance coils, that its temperature variations are negligible.

The resistances to be given to the leads and coils are determined by several considerations. The resistance between the galvanometer and a clip, or slider, must be so great that the unknown resistance introduced at the contact is negligible, while the sensitiveness of the apparatus is improved by making these resistances low, and winding the galvanometer to correspond according to a certain law. The resistances employed in this instrument were considered to be sufficient for the purpose for which it was designed, but for more accurate work than is required in measuring cables and dynamo-armatures, higher resistances should be used. The leads G H are 4 ohms or more for measurements of quantities most likely to occur in practice: when the resistance is only 1·333, the $\frac{1}{3}$ holes being plugged, care must be taken to make a sound connexion at H. The values of the leads G S are known when we have determined G H, and the resistance we propose to balance against the scale wire.

96.] ii. What is to be the form of the clips H is only a question of convenient attachment to the cable or dynamo.

The form shown is convenient for some purposes, while the best plan for others will be to lash the end of the lead to the conductor with tinned binding wire.

97.] iii. The section of the scale wire is determined by the amount of testing current required to produce readable deflections of the galvanometer-needle, when the balance is in error by an amount corresponding to the degree of accuracy with which it is required to measure. It is a question purely of the sensitiveness of the galvanometer, and is determined by a calculation of the electromotive force available at the galvanometer terminals. In Appendix II the following formulae are shown. When r, ρ are the resistances of the connexions as shown in Fig. 41, g the resistance of the galvanometer, P' the resistance of the small portion of the scale wire by which one of the sliders is out of adjustment, the electromotive force at the galvanometer terminals is ϵ, where $\epsilon = \frac{rg}{2r\rho + g(r+\rho)} \times P' \times$ the testing current in ampères along that part of the scale wire which lies between the sliders.

The value of g that gives the maximum value to the deflecting force is $\frac{2r\rho}{r+\rho}$, and when the galvanometer is wound to this resistance $\epsilon = \frac{r}{2(r+\rho)} \times P' \times$ testing current in ampères. From this it is seen that with a given galvanometer, and the same set of resistances plugged in the commutators, an error of adjustment of one slider by a certain small length of the scale gives the same galvanometer deflection, whatever be the distance between the sliders.

We proceed to calculate the electromotive forces at the galvanometer terminals due to errors of adjustment of one scale division, for each of the four arrangements of the plugs, viz. in holes marked 9, 3, 1, and $\frac{1}{3}$. In all cases the

value of P', which is the resistance of one division of the scale wire, is the same, viz. ·000175 ohm. Writing C for the current in ampères between S, S, or H, H.

(i) When the plugs are in holes marked 9, we have

$$r = 12, \quad \rho = 2{\cdot}333, \quad P' = {\cdot}000175\,;$$

then the best value for g is 3·9, and this gives

$$\epsilon = {\cdot}000073 \times C.$$

If the galvanometer were wound to 5 ohms, the value of ϵ would be ·00082 × C.

(ii) When the plugs are in holes marked 3, we have

$$r = 4, \quad \rho = 2{\cdot}333, \quad P' = {\cdot}000175\,;$$

then the best value for g is 2·94, and this gives

$$\epsilon = {\cdot}000055 \times C.$$

If the galvanometer were wound to 5 ohms, the value of ϵ would be ·00007 × C.

(iii) When the plugs are in holes marked 1, we have

$$r = 4, \quad \rho = 7, \quad P' = {\cdot}000175\,;$$

then the best value for g is 5·1, and this gives

$$\epsilon = {\cdot}000032 \times C.$$

(iv) When the plugs are in holes marked $\frac{1}{3}$, we have

$$r = 1{\cdot}333, \quad \rho = 7, \quad P' = {\cdot}000175\,;$$

then the best value for g is 2·24, and this gives

$$\epsilon = {\cdot}000014 \times C.$$

If the galvanometer is wound to 5 ohms, the value of $\epsilon = {\cdot}000018 \times C$.

The apparatus is designed primarily to deal with resistances from ·04 to ·013 ohm, and accordingly the galvanometer is wound to a resistance most suitable for that purpose, viz. 5 ohms. Now the form of portable galvanometer shown in the drawing, when wound to 5 ohms, gives a deflection sufficient for a null method with an electromotive force at the terminals of ·00012 volt [1]. Consequently the currents required to give good deflections with an error

[1] A controlling magnet, not shown in the illustration, is used to increase the sensitiveness of the galvanometer.

of adjustment of the sliders of one scale division are as follows:

With resistances—

·36 ohm – ·12 ohm	c may be 1·5 ampères making ϵ, ·000123.
·12 ohm – 04 ohm	c may be 1·7 ampères making ϵ, ·000119.
·04 ohm – ·0133 ohm	c may be 4 ampères making ϵ, ·000128.
For resistances below ·013	c may be 7 ampères making ϵ, ·000126.

The scale wire must accordingly be able to carry a maximum current of 7 ampères with little heating, and a platinoid wire of 100 mils. satisfies this requirement if the battery be only connected for the short time necessary to observe the movement of the galvanometer. By the rule of Article 83 the wire would carry 2 ampères continuously with a rise of temperature of 5° Ft. This current will allow resistances ·36 ohm to ·12 to be measured within ·0002, ·12 ohm to ·04 ohm within ·000075, ·04 to ·013 ohm within ·00005, and below ·0133 ohm within ·00003. This degree of accuracy is greater than is usually required in such work as the apparatus was designed for.

The movements of a reflecting galvanometer can be observed much better than those of a portable galvanometer, and where such an instrument wound to a low resistance can be used, considerations of sensitiveness are of small importance. In this case the values of r and ρ may be much increased, and a smaller testing current will be sufficient.

98.] iv. In the construction of the apparatus, the adjustment of the coils between G, G and the sliders are of course made after the slide wire is in place, as their resistances are proportional to that of the scale wire between the points 0 and 400, a quantity which cannot be accurately arranged beforehand.

99.] v. It has been pointed out that if the ratios of the resistances of the auxiliary conductors are accurately adjusted, it is immaterial what part of the scale wire is used for the measurement. It may, however, be noticed that the

effect of an error in one ratio can be reduced by making use of the property referred to in article 91, viz. that the exact value of the ratio $r' : \rho'$ is unimportant if the resistance bBCc is small. Suppose in Fig. 42 the ratio of $G_2D_2H_2 : G_2E_2S_2$ be in error: we can reduce the effect of this if S_2 be placed at the zero of the scale, and the resistance C_2H_2 be very small. Or if the ratio $G_1D_1H_1 : G_1E_1S_1$ be in error, the positions of the battery and the cable may be interchanged—which it is clear can be done through the symmetry of the arrangement —S_1 placed at the point 400, and the resistance of the lead from B_1 made very small.

100.] vi. The thick connexion between the main terminals which correspond to B_2C_1 in Fig. 42 is carried as close as possible to the scale wire, in order that the effects on the galvanometer of the currents along the two wires in opposite directions may counterbalance one another.

101.] The apparatus offers a convenient way of calibrating a wire. Placing it on the scale instead of the ordinary wire, and connecting a small resistance between C_1C_2, a balance is taken with the slides at different parts of the wire, and the distances between the sliders in the different positions of balance noted. The results obtained give the calibration required.

CHAPTER X.

THE COMPARISON OF HIGH RESISTANCES BY THE METHOD OF DIRECT DEFLECTIONS.

102.] WHEATSTONE'S bridge is not commonly used for measuring resistances of more than one megohm, because the high adjustable resistances necessary are very expensive, while the degree of accuracy required in such work is generally small, and can be secured easily by other methods. In many cases, moreover, the quantity observed is continually changing, and its successive values have to be observed: this cannot be conveniently done with a bridge.

103.] The highest resistances commonly measured in practice are those of such insulating substances as india rubber, gutta percha, and paraffin wax, used for the dielectrics of cables and condensers; and the quantities to be measured range from $\frac{1}{2}$ megohm to upwards of 30,000 megohms. These are examined by a method known as that of direct deflections. The two poles of a battery of high electromotive force, in series with a sensitive galvanometer, are connected to the conducting coatings on each side of the insulator to be examined. These coatings, in the case of a cable, are the central conductor on one side, and the sheathing, or the water of the tank in which the cable lies, on the other. In the case of a condenser they are, of course, the two sets of tinfoil sheets. The currents passing through the insulation from one of the conducting

coatings to the other, are estimated from the deflections of the galvanometer needle, and the values of these are determined by observing the deflection produced when a known resistance is placed in the circuit instead of the cable dielectric. This resistance should be of such dimensions as to be fairly comparable with the unknown to be determined. It is usually 1 megohm or $\frac{1}{10}$ megohm.

The arrangement of the connexions is shown in Fig. 44. G is the galvanometer, with its short circuit key *k*, and shunt box *s*. A drum of cable is shown in a tank of water

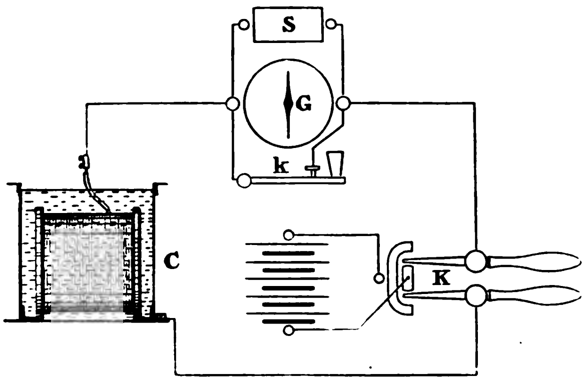


FIG. 44. Connexions for testing insulation by direct deflexions.

with its conductor connected to the galvanometer, and the tank to the Rymer Jones key K. K is so arranged that when both the levers are thrown over to the right the battery is connected in the circuit with its positive pole to the tank, and its negative pole to the galvanometer. If both levers are thrown to the left the battery is connected in the reversed sense. When the levers are thrown outwards in opposite directions, one pole of the battery is disconnected, and the cable and galvanometer are directly in circuit with one another. In no position of K can the battery be short circuited.

104.] The theory of the experiment is usually given thus.

Let E be the electromotive force of the battery and B its resistance. Let S be the resistance of the standard and R the unknown resistance we require to measure. Let G be the resistance of the galvanometer, Gs its resistance when shunted in dealing with S the standard; and Gs' its resistance when shunted in dealing with R. Then, when we have R in the circuit, the whole current is $E\{B+Gs'+R\}^{-1}$; and the part of it which passes through the galvanometer is $Es'\{B+Gs'+R\}^{-1}$. When we have S in the circuit, the whole current is $E\{B+Gs+S\}^{-1}$; and the part of it which passes through the galvanometer is $Es\{B+Gs+S\}^{-1}$. Suppose these currents give deflections on the scale of equal currents D, d: D in the case of the standard, and d when R is in circuit. Then, since the deflections are proportional to the currents, we have $\dfrac{s'(B+Gs+S)}{s(B+Gs'+R)}=\dfrac{d}{D}$. From this equation R can be determined, since all the other quantities are known.

In the case when B and Gs are small compared with S, and B and Gs' small compared with R, we may omit them from the calculation, and write $R=\dfrac{s'D}{sD}\cdot S$.

For an example, suppose G to be 7000 ohms, B to be 30 ohms, S to be 1 megohm: and suppose that in taking the deflection with the standard megohm in series we use $\frac{1}{999}$ shunt, and obtain a deflection of 123 divisions, and that in taking the deflection with the unknown resistance in series we use $\frac{1}{9}$ shunt, and obtain a deflection of 275 divisions. Then $Gs=7$, $Gs'=700$, $s=\frac{1}{1000}$, and $s'=\frac{1}{10}$. Then Gs is insignificant compared to S, and Gs' when compared to R. B is also a small quantity. Writing then $R=\dfrac{s'D}{sd}\cdot S$, we have $R=44{\cdot}7$ megohms.

105.] The galvanometer used is Lord Kelvin's reflecting instrument. A straight scale 15 inches long is placed at

a distance of about 43 inches from the mirror, and is divided into 600 equal divisions—i.e. 40 divisions to the inch. These are as fine as can be easily read with the naked eye. The scale subtends an angle of 20 degrees at the mirror, which allows deflections of the galvanometer needle within 5 degrees on either side of the central or zero position to be observed, the angular deflection of the reflected beam of light being double that of the magnetized needle. One division near the centre represents an angular deflection of the needle of one minute of arc. The scale is a scale of tangents, and as these instruments are approximately tangent galvanometers, the scale is sufficiently nearly one of equal currents for the measurement of insulation resistances. Where an accurate scale of currents is required, the scale must be calibrated, or else only a very small portion of it used. A correction may be applied to the readings on one side of the zero, sufficient for some purposes, by setting the scale at some angle to the line joining the zero to the mirror other than a right angle. This of course increases the error on the other side of the zero, so that all the readings must be taken in one direction, by reversing the connexions of the galvanometer with the sense of the current. A galvanometer of this kind, wound to 7000 ohms, may readily be adjusted to give a deflection of one division with $\frac{1}{1000}$ microampère, and can be made much more sensitive than this if required. Thus, with a battery of 100 volts, we can obtain deflections readable within 1 per cent. through external resistances of 1000 megohms, and the limit of possible range is then by no means reached.

106.] Suppose the apparatus to be in the condition shown in Fig. 44, ready to begin testing. The first operation is to throw both the handles of K to the right, thereby connecting the negative pole of the battery to the conductor of the cable through the galvanometer, and the positive pole to the tank. Immediately this is done two quantities of

electricity of opposite signs pass from the battery to the two sides of the dielectric, charging them to the difference of potential of the battery. The amount of this charge, measured in microcoulombs, is the product of the capacity of the cable in microfarads, and electromotive force of the battery in volts. This current is so considerable compared with the small succeeding currents, which the galvanometer is arranged to measure, that it is necessary to close the short circuit key while it is passing. It lasts but a very short time however, even in long cables, and *k* may be opened perhaps 10 seconds after the connexion of the battery. Then is observed a deflection of the galvanometer, which continually decreases, rapidly at first, then more and more slowly, and finally approaches a minimum or asymptotic value. It sensibly takes this value some 30 to 60 minutes after the first application of the battery. During this period it is usual to record the galvanometer deflection at the completion of every minute.

This first operation is called testing with a negative current, and is continued for some fixed time, usually 15 or 30 minutes.

On the completion of the last minute the second part of the test is begun. Key *k* is closed so that the galvanometer is short circuited and the deflection brought to zero. The left-hand lever of K is then thrown over to the left, so that both levers rest on the central contact, and one pole of the battery is disconnected, these operations being performed as rapidly as possible. The two sides of the dielectric are thereby connected together, and their charges unite and neutralize one another. The quantity of electricity passing from one side to the other is the same as passed originally when the cable was charged, and is so considerable as to require the galvanometer to be short circuited. After allowing a few seconds to elapse for this discharge, *k* is again opened.

A much smaller deflection is now observed than before, and on the opposite side of the zero. This deflection, like the former, decreases, but at a continually slower and slower rate, tending to a minimum or asymptotic value, which in this case is zero. It is not usual to continue this part of the test for more than 5 minutes, when the deflection has still a very appreciable value. The scale readings during this period are called the earth readings, and are recorded every minute.

The last operation of the test is now begun. The galvanometer key *k* is again closed, and the right-hand lever of K thrown to the left. By this the testing battery is connected to the circuit with the positive pole to the cable conductor, and, as before, large charges of electricity pass to the two coatings of the dielectric, charging them again to the difference of potential of the battery, but in an opposite direction to the first charge. On opening the short circuit key *k*, a deflection is observed on the same side as those of the second operation, and on the opposite side to those of the first part. This deflection, like the others, gradually falls, rapidly at first, and tends more and more slowly to a minimum value. This is called testing with a positive current, because the positive pole of the battery is connected to the cable conductor, and the values of the deflections are recorded every minute. The observations may be continued 5 minutes when the test is completed.

107.] The following figures are those of an actual test on a completed length of 346 nautical miles of a cable with gutta percha insulation.

The battery power was 300 Leclanché cells, equivalent to about 450 volts.

The resistance of the galvanometer was 6823 ohms.

The standard resistance used was 1 megohm, and in observing the deflection of the galvanometer, with this in

series, a shunt of 30 ohms was used; with the cable in circuit the shunt was 900 ohms.

The cable had lain for a long time previously with its conductor connected to the sheathing, so it was as nearly as possible free from the charge left by a previous test. The battery was applied first with the zinc or negative pole to the conductor.

The deflections observed were as follows:—

Time after the connexion of the battery.	Deflection observed.
1 minute	299 divisions to the right.
2 "	254 " "
3 "	237 " "
4 "	229 " "
5 "	223 " "
6 "	219 " "
7 "	216 " "
8 "	214 " "
9 "	212 " "
10 "	210½ " "
11 "	209 " "
12 "	207½ " "
13 "	206 " "
14 "	205 " "
15 "	204 " "
16 "	203 " "
17 "	202 " "
18 "	201 " "
19 "	200 " "
20 "	199 " "
21 "	198½ " "
22 "	198 " "
23 "	197½ " "
24 "	197 " "
25 "	196½ " "
26 "	196 " "
27 "	195½ " "
28 "	195 " "
29 "	194½ " "
30 "	194 " "

At this point the left-hand lever of the key was thrown

on to the middle contact so that the battery was disconnected, and the circuit now consists of the cable and galvanometer only. These deflections are called the earth readings.

Time after disconnecting the battery.	Deflection observed.
1 minute	102 divisions to the left
2 "	62 " "
3 "	48 " "
4 "	40 " "
5 "	35 " "

At the completion of the 5th minute of earth readings, the right-hand lever of K is thrown to the left, so that the battery is again placed in the circuit: this time in the reverse direction, with the positive or carbon pole connected to the conductor of the cable. The following were the deflections observed:—

Time after re-connecting the battery.	Deflection.
1 minute	334 divisions to the left
2 "	287 " "
3 "	269 " "
4 "	259 " "
5 "	253 " "

This concluded the test. The galvanometer was then short circuited and the battery disconnected. The resistance of the cable is calculated from the formulae already given on page 127, the simplified formula being nearly always applicable, viz. $R = \frac{s'D}{sd} . S$. This makes the value of R at the end of the first minute after the application of the battery 27·24 megohms, and at the end of the 30th minute 42·1 megohms. These are the resistances of the insulation of the whole 346 nautical miles, and, to obtain the insulation of 1 nautical mile from these figures, they are multiplied by 346 the length of the cable. This gives the insulation resistance per nautical mile as 9,420 megohms after 1 minute's test, and 14,560 megohms after 30 minutes' test.

108.] Care must be taken in making this test that no current passes through the galvanometer except that which also passes through the cable insulation. The principal places where leakage is likely to occur are at the galvanometer itself, its shunt box and short circuit key, and at the ends of the cable. On looking at Fig. 44, on page 126 it will be seen that if there is any connexion between the galvanometer coils and earth, some current will pass that way, and produce a deflection on the galvanometer which does not represent any current passing through the cable insulation. On the other hand, if the end of the core of the cable be dirty, a current will pass from the conductor to the sheathing over the dirty surface, and will apparently show the insulation of the cable to be less than its real value.

The existence of the first defect, viz. leakage at the galvanometer or its leads, is tested by making a preliminary trial exactly as shown in the diagram, but with the conductor of the cable disconnected from its lead. The deflection of the galvanometer then observed is noted on the test sheet, and if it amounts to more than one or two divisions its cause should be discovered and put right. The second defect may be avoided by carefully trimming the ends of the core with a clean and sharp knife before beginning to test, but its existence cannot be directly tested. It will probably show itself in the course of the main test by irregularities in the fall of the deflections.

109.] The condition of the dielectric is estimated partly from the actual value of the resistance thus obtained, and partly from the steadiness with which the changes in the values of the deflections take place. This gradual decrease of the deflection measures what is termed the electrification of the cable, and a steady electrification, such as is shown in the above example, is considered a very favourable sign. If the curves drawn in Fig. 45, which are plotted

from the observed values, showed sharp changes of curvature, and irregularities, it would be recognized that either the ends of the cable were wet and dirty, or that the cable itself was faulty.

110.] It is seen that what we have called the resistance of the insulation, and have defined as the ratio of the electromotive force applied to the current that passes, is a very variable quantity, depending on the length of time that has elapsed from the first application of the battery, and also on any reversals of its direction that have taken place. Engineers' specifications, drawn for practically testing such insulated cables, usually stipulate that the insulation resistance shall be within certain limits one minute after the battery is connected, or, as it is commonly termed, after one minute's electrification; but it does not appear that one minute serves better than any other time for the practical comparison of the substances used as insulators, and the condition has no scientific basis whatever. This, of course, does not in any way affect its practical utility. Cases have occurred in which the insulation resistance of a cable has been specified in a contract without any mention of the time to elapse between the connexion of the battery and the critical observation; and such an omission—especially in the case of india rubber, in which the resistance, as specified above, changes more than in gutta percha—may render the clause of the specification valueless.

111.] To determine if the results of the method of direct deflections show the existence in a dielectric of a physical property analogous to the resistance of a metallic conductor, we may define again the exact meaning of the word resistance, and then examine the figures given in article 107. The energy expended in passing a current of electricity through a conductor may appear in several different forms. The first is the heating of the conductor, and is measured by the square of the current multiplied by a quantity called

the resistance of the circuit. Others are electrolytic action, changes in the neighbouring electrostatic or magnetic fields, heat absorbed or given out at the junction of dissimilar metals. Now in the particular case when no energy is expended on any of these latter forms, and the whole applied energy appears as heat in the conductor, the resistance becomes the ratio of the current to the electromotive force at the terminals. It is found by experiment that this is the case with homogeneous, linear, metallic conductors, and the resistance of these is defined in those terms.

To put the matter in symbols, let E be the electromotive force applied to the conductor, C the current passing, R the resistance. Then EC is the total energy supplied, and C^2R the heat developed in the conductor. Let W be the energy expended in the circuit otherwise than in heating it. Then

$$EC = C^2R + W, \text{ and } R \text{ only equals } \frac{E}{C} \text{ if } W = 0.$$

Consequently before the resistance of a circuit can be measured by the ratio of the electromotive force applied to the current that passes through it, it is necessary to prove that the energy is expended in no other form than in heating the conductor. The particular case where some of the energy W appears as heat at the junction of dissimilar metals is distinguished by the reversal of the phenomenon with the sense of the current; i. e. if the current be reversed heat will be absorbed at the junction instead of being developed.

It may be remarked that this statement is in no way dependent on Ohm's law, which merely states that in a metallic conductor R is independent of E or C, a theorem which might be disproved without affecting our argument.

We are, therefore, not justified in calling the ratio between the electromotive force applied to the dielectric of a cable, and the current that passes from the battery, the resistance of that dielectric, unless we either ascertain that the whole

of the energy of the current is converted into heat, and that equally in whichever direction the current is applied; or

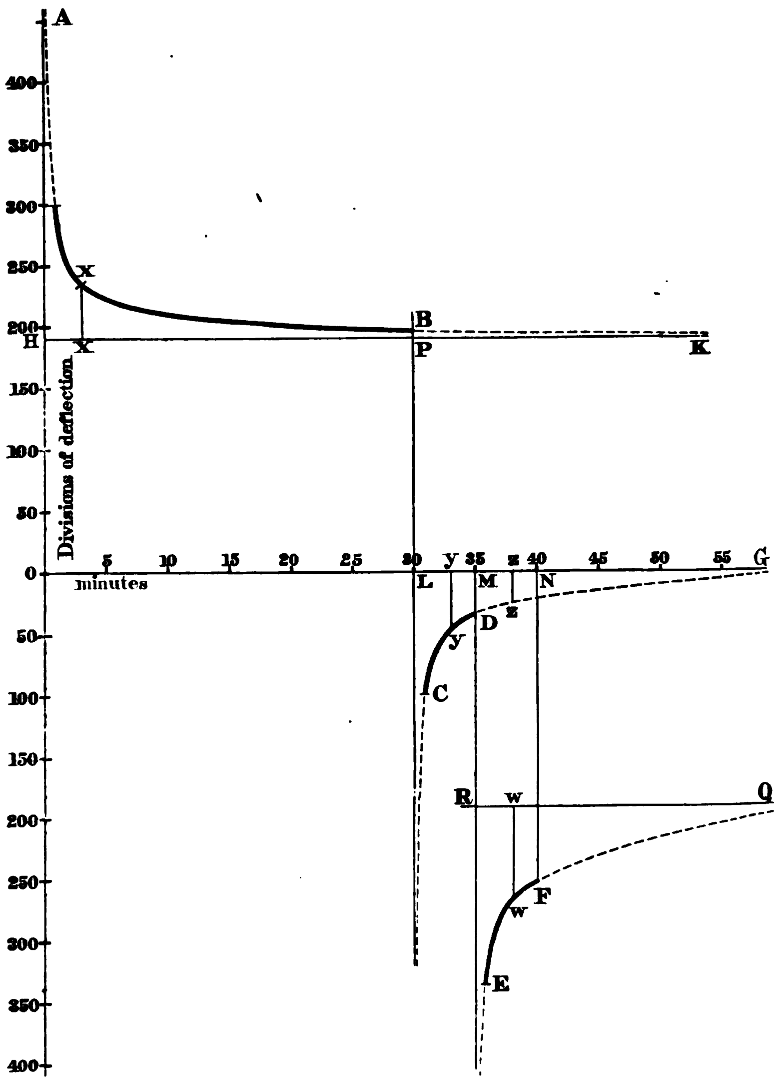


Fig. 45. Curve plotted from the observations of an insulation test.

else agree to use the term resistance, when applied to

dielectrics, in a special and peculiar sense. As a matter of fact, the last alternative has been adopted, but little attention is paid to the peculiar use.

112.] For the discussion of the phenomena of the method of direct deflections, the figures of the test given on pages 131, 132 are plotted in curves in the diagram of Fig. 45; the abscissae represent minutes of time, the ordinates represent the galvanometer deflections proportional to the currents.

The values of the ordinates immediately after the changes of the battery key are unknown, since the currents are at once so violent, and increase or diminish so rapidly, that the ordinary instruments are unsuited for following them; but the fact that a ballistic galvanometer gives equal deflections on the charge and discharge of a cable shows that the areas enclosed by these portions of the curves are all equal.

The curve AB represents the first 30 minutes of the test when the negative pole of the battery is connected to the conductor. The curve CD represents the 5 minutes of earth readings: the curve EF when the positive pole is connected.

The dotted lines show what the extensions of these curves would have been had the different parts of the tests been continued. Their character is well known from long experiments on similar cables.

The following points are to be noticed: —

(i) The curve AB has a horizontal asymptote HK at a distance of about 190 divisions from the axis.

(ii) The curve CD has an asymptote which is the horizontal axis.

(iii) The curve EF has a horizontal asymptote RQ at the same distance from the axis as HK, viz. 190 divisions.

(iv) The area HABP is equal to the area LCDG within small errors.

(v) The area REFQ is equal to the sum of the areas MDG and HABK.

These equalities may be seen by comparing the ordinates intercepted between the three curves and their asymptotes, at the same intervals of time after their commencement. Thus to compare the areas HABP and LCDG take an ordinate of AB three minutes from O, and an ordinate of CD, three minutes from L. We have $xx = 47$, and $yy = 48$, which are nearly equal; and all the ordinates thus taken in pairs will be found approximately equal, and the areas of the two curves also equal. To compare REFQ with the sum of MDG, and HABK, we have $ww = 79$, $xx = 47$, and $zz = 30$; so that $ww = xx + zz$ very nearly.

113.] These coincidences are sometimes found to be remarkably exact, especially when testing great lengths of cable which have been long in the same circumstances and position without any change of temperature. Many observations on the various properties of these insulators show that they require a very long time to complete any change after an alteration of their circumstances. For instance, in the case of a cable carried in a ship's tank from England to Cape Horn, on the way to Peru, the temperature of the tank water rose steadily on the voyage into the tropics, and fell again on proceeding further south. These changes were fairly closely followed by the copper conductor, as calculated from its measured resistance, while the condition of the gutta percha as determined by its insulation resistance lagged behind many days, and probably never reached its final value for the hottest region at all. Unless a cable has been lying for a long time, a very close agreement of the values of the deflections with the equalities pointed out is not to be expected.

114.] The electricity passed into the cable from the battery during the first 30 minutes of the test may be divided into two parts; one part, represented by the area OHPL, is that

due to a steady current of electricity OH; the other part, represented by the area HABP, is due to a rapidly falling current. The energy of the electricity OHPL is not recoverable in that form on discharging the cable, and may be assumed to have been converted into heat. The energy of the electricity HABP is wholly recoverable, and, in the absence of any evidence of electrolytic or thermo-electric action, probably exists as a stress in the dielectric. This part of the experiment suggests that the electrical energy passing into the cable may be divided into two parts, one of which is converted into heat, and is not recoverable as electric energy, the other of which is recoverable. The current OH conveys the first part, and the ratio of the applied electromotive force to this current may properly be called the resistance of the dielectric; and the resistance of the cable calculated from the lowest, or asymptotic value of the deflection, is the quantity really analogous to the resistance of a metallic conductor. Its value is exceedingly sensitive to changes of temperature, but whether, when at a fixed temperature, it is subject to Ohm's law is not known to the author.

115.] The greater proportion of the part HABP passes into the cable within a very short time after the connexion of the battery, and is recognized as being the charge taken up by the cable in virtue of its electrostatic capacity; but the remainder of it is generally treated as if it were due to an independent phenomenon, which is called electrification, or absorption. Now, in the absence of any reason for attributing the quantity to two distinct causes, it would seem more reasonable to treat it—at least provisionally—as due to a single cause. Moreover, no rule has been proposed for dividing the two parts in any particular way. Probably the whole is one phenomenon with the main electrostatic charge, and though cables and condensers take and give up again very rapidly indeed the greater part of their charges,

the process is not completed instantaneously, but continues for a very long time at a decreasing rate. Indeed, when once a submarine cable has been charged, it will show signs of a small residual charge after having been continually short circuited for many months. These solid dielectrics, when subjected to electrostatic strain, do not undergo their total stress at once, but only show it after a long period of time, and require a corresponding time to lose it. Some further considerations may be offered in support of this view. If the fall in the current during an insulation test were due to an opposing force of electrolysis, we should expect to find some of the chemical products of such an action, and these have never been detected. Moreover, all glass exhibits precisely the same phenomenon of slow absorption and slow discharge, in addition to the sudden electrostatic charge and discharge, while, at least with some kinds, it is quite incapable of transmitting any permanent current. The explanation of absorption must in this case be looked for in the electrostatic strain, and not in the passage of an electrical current.

115 a.] Two hypotheses have been offered of the physical structure of the dielectric to explain the phenomena. Clerk Maxwell showed that they might be explained by supposing the dielectric to consist of materials of different specific conductivities and electrostatic inductive capacities, but offered the suggestion as an illustration of the phenomena rather than as a theory of the actual structure of the dielectric[1].

Messrs. Mascart and Joubert[2] suggest that an explanation is to be sought in a form of polarization analogous to that of a magnet, and the author has pointed out that if small elements, bearing opposite electrostatic charges at their

[1] Clerk Maxwell, *Electricity and Magnetism*, vol. i. § 330.

[2] *A Treatise on Electricity and Magnetism* by Mascart and Joubert, translated by E. Atkinson, vol. i. p. 95.

ends, similar to crystals, be substituted for the magnetic elements in Ewing's theory of the magnet, this suggestion would be realized[1]. The phenomena of absorption would require that some of the electrified elements should move in a viscous medium. Either Clerk Maxwell's hypothesis, or this, explains the very curious property possessed by these dielectrics of being able to absorb successively two or more charges in opposite senses, and to return them in the reversed order, and of their proper signs.

[1] *The Electrician*, Feb. 5, 1892, p. 355.

CHAPTER XI.

Other Methods of Measuring High Resistances.

Very high resistances may be dealt with in several other ways.

116.] If a known resistance large enough to be fairly comparable with that to be measured is available, the comparison may be made by placing the two resistances in series with a battery, and comparing the differences of potentials at their ends. Thus if, in Fig. 46, AB be the

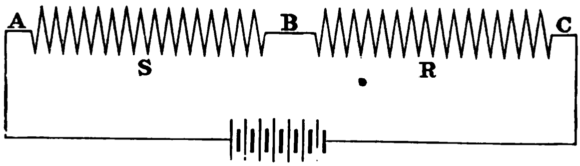


Fig. 46.

known resistance S, BC be the unknown R, and A, B, C be the potentials of those points, we have from the definition of resistance $\frac{A-B}{S} = \frac{B-C}{R}$, and $R = \frac{B-C}{A-B} \cdot S$.

The measurement of $\frac{B-C}{A-B}$ may be made in several ways.

117.] i. We may determine $A-B$ and $B-C$ separately by connecting first AB, then BC, to an electrometer, and compare the readings.

118.] ii. We may charge a suitable condenser from the

points A, B, and then discharge it through a ballistic galvanometer. The operation is then repeated with the points B, C, and the two galvanometer deflections compared. The instruments may be connected as they are shown in Fig. 47. A condenser whose capacity is K is connected between the point A and a key *k*, through which, by moving the levers to the right, it is charged to the difference of potential that exists between A and B. On throwing the levers of *k* over to the left, the condenser is discharged through the galvanometer, shunted if necessary, and the deflection of the needle

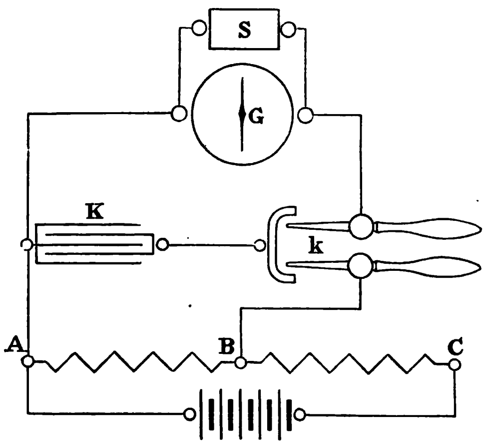


FIG. 47.

noted. Let it be D. K is then connected to B, and the lever of *k* to C, and the experiment repeated. Let the deflection now be *d*. Suppose that G is the resistance of the galvanometer, G*s* its resistance in parallel with the shunt for observing D, and G*s'* for observing *d*. Since the galvanometer is ballistic, the deflection on the scale is proportional to the quantity of electricity that passes through it, and if the galvanometer be shunted so that the resistance is G*s*, and a quantity of electricity Q be passed through it in parallel with its shunt, Q*s* will be the quantity that passes through the instrument, and (Q I − *s*) through the shunt.

The charge of the condenser when connected to AB is (A − B) K, and the amount of electricity that passes through the galvanometer will be consequently (A − B) Ks; so the amount passing through the galvanometer when the condenser is charged from BC, is (B − C) Ks'. These quantities are proportional to the deflections,

$$\text{i.e. } \frac{(A-B)s}{(B-C)s'} = \frac{D}{d}; \text{ but } \frac{A-B}{B-C} = \frac{S}{R}; \text{ hence } R = \frac{ds}{Ds'} S.$$

Thus, for example, if S be one megohm, $s = 1$, $s' = \frac{1}{1000}$, $d = 255$, and $D = 106$, we should have, R = 2,400 megohms.

If the resistance is very high, the time required to charge the condenser is long and the method becomes tedious. For example, nearly twenty hours are required to charge a condenser of one microfarad within ·1 per cent. of its full capacity through a resistance of 10,000 megohms. Moreover, the fact that the insulation of the condenser is not perfect may have to be considered.

118 a.] A ballistic galvanometer when used with a shunt box is liable to a particular error. The galvanometer coils have a considerable inductance, and when the current of electricity from a discharging condenser is started through them, a resisting electromotive force, due to the growth of a magnetic field, prevents the current rising immediately to the value determined by the ratio of the electromotive force acting in the circuit to the resistance. The total induction of the coils tends to a maximum value, and afterwards decreases; and while its development constitutes an electromotive force in opposition to the current, its after disappearance constitutes an electromotive force assisting it. Consequently during the earlier part of the discharge, the current through the galvanometer is less than that measured at any moment by the ratio of the values of the difference of potential of the condenser plates and the galvanometer resistance, is equal to it when the point of maximum induction is reached, and is greater afterwards.

The shunt circuit being non-inductively wound is not subject to this action, and the current conveyed by it is throughout proportional to the difference of potential of the condenser plates.

The total quantity of electricity conveyed by the galvanometer bears to that conveyed by the shunt the inverse ratio of their resistances, but in the earlier stages of the discharge the galvanometer current is in defect, in the later stages in excess, of its proper proportion. A proof of this is given in Appendix IV.

The needle of an ideal ballistic galvanometer would be deflected to exactly the same extent by the discharge of a given quantity of electricity, whatever might be the arrangement in time of the various portions of the discharge, but this is not the case in practical instruments, and a reading only accurately measures the electricity that passes, if the whole discharge takes place before the needle appreciably leaves its position of maximum sensibility. Any action which extends the duration of a large part of the discharge till the needle has left its central position diminishes the total deflection.

Thus in comparing condenser discharges by means of a ballistic galvanometer, although the galvanometer and shunt circuits share the discharge in inverse proportion to their resistances, yet the deflection of the galvanometer needle is reduced by the self-induction of its coils, and the reduction may be considerable under favourable conditions.

119.] iii. Another way of comparing two such resistances as these is a modification of Wheatstone's bridge, made by using a condenser and galvanometer instead of a galvanometer alone. The arrangement of the apparatus is shown in Fig. 48. Here a condenser is charged to the difference of potential of the points PQ, by moving the levers of the key to the left, and then discharged through a galvanometer by moving them to the right. If no deflection of the

galvanometer is observed P and Q must be at the same potential, and we have $\frac{R}{S} = \frac{A}{B}$, i. e. $R = \frac{A}{B} . S$. There is practically no limit to the small difference of potential

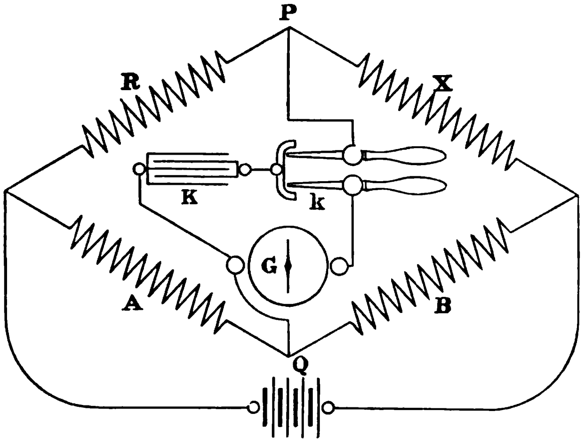


FIG. 48.

between P and Q which can be detected in this way, provided the capacity of K be large enough, and the action is quite independent of the dimensions of R, S, A, B provided sufficient time be allowed for charging the condenser through high resistances. If a galvanometer were connected in the usual way between P and Q, the current through it due to a slight error of balance would be so small if R and S were very large, that the arrangement would be insensitive.

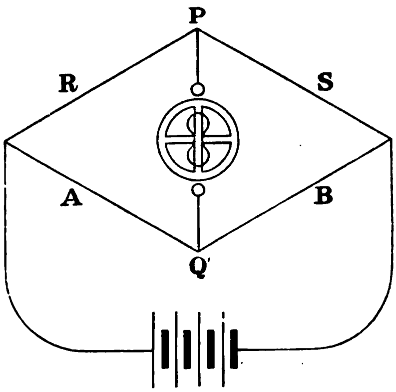


FIG. 49.

Of course the equality of the potentials at P and Q may be

tested by connecting them with an electrometer as in Fig. 49. The sensitiveness is independent of the dimensions of R and S equally with the last arrangement, but an electrometer is a much less delicate instrument for the detection of small differences of potential than the combination of condenser and galvanometer.

For either of these methods a set of Kelvin and Varley's slides may be conveniently used for the arms of the bridge A and B.

120.] Major Cardew, R.E., has proposed to compare the potentials at A B C by connecting A and C to the quadrants of a quadrant electrometer, and B to the vane as in Fig. 50, a battery of very high electromotive force being used. S is to be adjusted till there is no deflection, when we have A − B = B − C, and R = S.

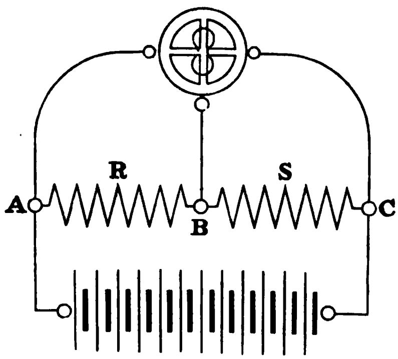


FIG. 50.

121.] For most practical applications all these methods require standards of very high resistance, while the last described requires the standard to be adjustable; and such resistances are rarely available. The following do not require the use of a standard resistance at all, but depend on a determination of the rate at which a charged condenser loses its charge, when the two sides are connected together through the resistance to be measured. They are known as loss of charge methods.

In carrying them out the condenser is first charged from a battery, and then connected to the terminals of a resistance. At the end of a given time it is discharged through a ballistic galvanometer and the reading observed. The experiment is then repeated, except that the terminals of

the condenser are insulated for the same period of time instead of being connected to the resistance. Let K be the capacity of the condenser in microfarads, R the resistance to be measured in megohms, t the length of time in seconds between the charging and discharging of the condenser, D the deflection of the ballistic galvanometer after t seconds' insulation, and d after t seconds' connexion to the resistance. Then we have $\mathrm{R\,K}\log\frac{\mathrm{D}}{d}=t$, from which R may be found. (For the proof of this see Appendix IV.)

The connexions of the instruments are shown in the diagram. K is the condenser, B the charging battery, R the

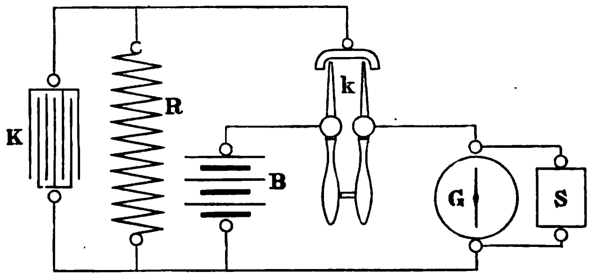


FIG. 51.

resistance to be measured, and G the ballistic galvanometer. The first operation is to throw the levers of the Rymer Jones key k to the right, so as to charge the condenser to the difference of potential of B, and as soon as it is charged the left-hand lever of k is thrown to the left. The condenser then begins slowly to discharge itself through the high resistance R. When this has continued for t seconds, the right-hand lever is moved to the left, and all electricity left in the condenser discharged through the galvanometer. The deflection thus obtained is d. R is then disconnected and the operations repeated, the deflection obtained being D. By allowing the same time to elapse between the charge and the discharge in the second part of the test when the

condenser is insulated as in the part when it is connected to R, we eliminate from the equation any terms depending on imperfect insulation in the condenser or its connexions. A box of shunts is shown connected in the diagram; if the same shunt is used in reading both discharges—and these will not generally be widely different—its coefficient S does not appear in the equation; any change of shunt necessary must be considered in estimating the value of $\frac{D}{d}$.

To take an example let us suppose we require to measure the resistance of a short piece of gutta percha core whose capacity we may neglect compared with K. K is ten microfarads, the observed value of D 195 divisions, d is 102 divisions, t is 300 seconds. Then $R \times 10 \times \log \frac{195}{102} = 300$, i. e. $R = 106$ megohms.

In making this test the value of K and the electromotive force of the battery should be chosen so as to give a convenient value to D, and t should be chosen so as to make d about one half the value of D.

122.] With an electrometer the loss of charge may be measured by observing the fall of the potential of the

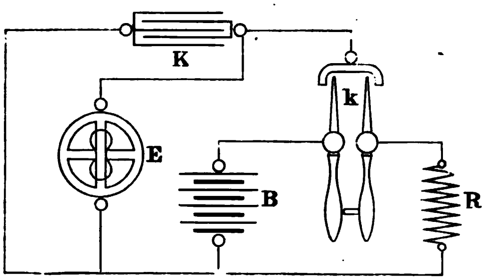


FIG. 52.

condenser, and the resistance is obtained by comparing the two electrometer readings, one taken t seconds after charging when the condenser has been connected to the resistance,

and the other taken after it has been merely insulated for the same length of time.

If as before R is the resistance in megohms, K the capacity in microfarads, t the time in seconds, D the electrometer reading after t seconds' insulation, d after t seconds' connexion to the resistance, we have as before $\text{RK} \log \frac{\text{D}}{d} = t$. The connexions in this case are shown in Fig. 52.

123.] The capacity of a submarine cable is so considerable that it is unnecessary to use any other condenser, and the insulation resistance may be measured by observing the loss

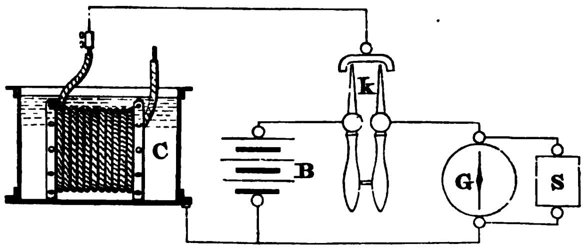


Fig. 53.

of charge of the cable itself. The resistance is obtained from a comparison of the charge originally given to the cable, with that remaining in it, after insulation for some given length of time, the charge that is lost during that time being assumed to have passed through the insulation. It is necessary to take great care that the ends of the cable are cleanly cut, as it is not possible to determine by a separate experiment whether any of the loss is due to leakage over the surface of the insulation from the exposed conductor.

Let K be the capacity of the cable in microfarads, R the insulation resistance in megohms, D the deflection observed on a ballistic galvanometer when the cable is discharged immediately after charging, d the deflection observed when the cable has been insulated for t seconds between charging

and discharging. Then we have $KR \log \frac{D}{d} = t$. (See Appendix IV.)

The arrangement of the apparatus is shown in Fig. 53. The cable is shown on a drum in a tank of water, the end of the conductor being connected to a Rymer Jones key, whose levers are connected to the galvanometer and battery respectively. The other side of the galvanometer, the other pole of the battery, and the outer surface of the cable, i. e. the tank of water, are connected together.

The operations are as follows:

(i) Both levers of the key k are thrown to the right, thus charging the cable to the difference of potential of the battery.

(ii) Both levers are thrown quickly to the left, thus discharging the cable through the galvanometer before it has had time to lose any of its charge. The deflection observed is D.

(iii) The cable is again charged by moving both levers to the right.

(iv) The left-hand lever is moved to the left, thus disconnecting the battery, and insulating the cable. The key is left in that position for t seconds.

(v) The right-hand lever is moved to the left, and the cable again discharged through the galvanometer. The deflection observed is d.

For an example, a cable whose capacity is 60 microfarads, gives on discharging immediately after it is charged a deflection of 241 divisions. After recharging and insulating for 300 seconds, it gives a deflection on discharge of 148 divisions. We have, since $KR \log \frac{D}{d} = t$,

$$60 \times R \log \frac{241}{148} = 300; \text{ i. e. } R = 23{\cdot}5 \text{ megohms.}$$

If we use an electrometer instead of a galvanometer, it is connected directly to the conductor of the cable, and the deflection when the battery is connected is compared with the deflection observed t seconds after it has been disconnected.

CHAPTER XII.

THE RESISTANCES OF BATTERIES AND ELECTROLYTES.

124.] THE form of electrolytic cell which most commonly occurs in practice is a battery, and the principal methods employed for the measurement are the following:

(*a*) Mance's method.
(*b*) Beetz' method.
(*c*) Kohlrausch's method.

These will be described in order.

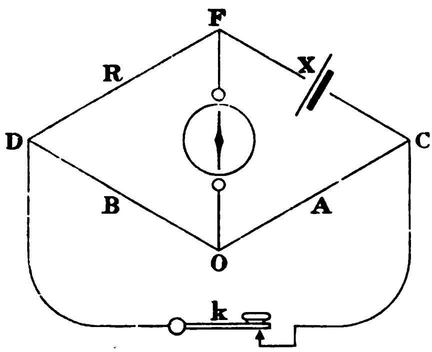


FIG. 54.

125.] (*a*) Sir Henry Mance's method consists in placing the battery whose resistance we desire to measure in one side of the quadrilateral of a Wheatstone's bridge, the ordinary testing battery being replaced by a make and break key, as shown in Fig. 54, where the letters represent the resistances of the conductors. Then it may be shown

that, if $X \times B = A \times R$, the current through FO is independent of the value of the resistance in CD. Of course with this arrangement there is always a current passing through FO, and the galvanometer shows a permanent deflection; but if the relative resistances of A, B, and R are such that no change in the deflection of the galvanometer needle is observed when large alterations are made in the resistance of CD, then $B \times X$ must equal $A \times R$, and X the resistance of the battery $= \frac{A}{B} R$.

In making this test, the resistance of CD is changed repeatedly from infinity to a very small quantity by closing and opening the key k, and the resistance of R is adjusted until the galvanometer needle remains steady during the process.

Since a galvanometer needle is most sensitive to small changes in the magnetic field of its coils when it is in its central position, it is usual to balance the action of the permanent current through the coils by some external adjustable arrangement such as a controlling magnet, or a subsidiary coil carrying an independent current, and thereby bring the pointer to zero. The latter is the more convenient method. Indeed, such a coil, through which a current in either direction and of any desired strength can be sent at will from an auxiliary battery, may be advantageously fitted to every reflecting galvanometer, and employed to check the swing as required. If this latter purpose alone is in view, the coil may be connected in series with a second near the operator's hand. A slight motion of a permanent magnet passed through the middle of this latter coil is quite sufficient to check the swing of the needle.

As to the proportions to be given to the other resistances of the bridge for most advantageously making this test, it is shown in Appendix V, on page 195, that the resistances

of the galvanometer and of B should equal that of the battery, and the resistances of A and E should be respectively twice and one-half that of the battery. These rules may be employed as a guide in the case of Daniell's cells, which have a high resistance compared with their electromotive force and dimensions, and contain an active depolariser, but the resistances assigned would be much too low for measuring Leclanché cells.

126.] Mance's method may be conveniently modified by placing a condenser in series with the galvanometer in the way shown in Fig. 55. The galvanometer needle will then normally stand at zero, and will only be deflected when the charge in the condenser is altered by a change of the difference of the potentials at F O. This plan saves the use of any controlling magnet or subsidiary coil, and the resistance of the galvanometer need not be taken into consideration if the condenser be large enough.

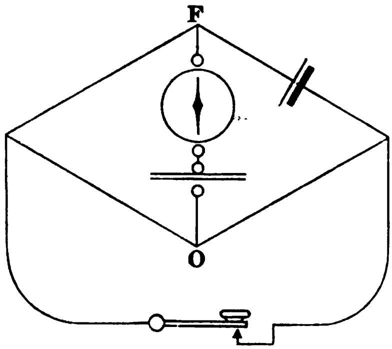


FIG. 55.

The resistance of an electrolytic cell, which is not itself a source of electromotive force, may be tested by placing it in series with a battery cell in one arm of the bridge. The sum of the resistances of the two is then determined, and that of the battery measured separately and subtracted from the first result.

127.] (*b*) Beetz' method consists in placing the battery in a closed circuit, and then determining the resistance which is intercepted between two points whose potentials differ by some fixed amount. This amount is the electromotive force of another battery used for reference, and must be less than

that of the battery under examination. The values of the electromotive forces of the two batteries are eliminated from the expression for the resistance by making two experiments with different resistances in the main circuit.

The connexions are shown diagrammatically in Fig. 56. E is the battery whose resistance is to be measured, e the other battery used for reference. Then if E e be the electromotive forces of these, e must be chosen so as to be less than E. One electrode of e is connected through a galvanometer to a point A of the main circuit, and the other to

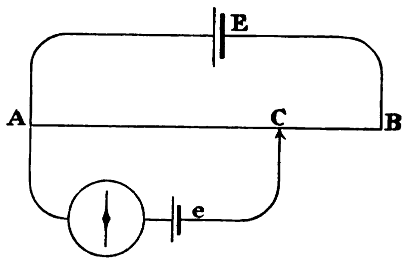


FIG. 56.

a point C found by trial such that when contact is made at C, no deflection is observed on the galvanometer. In other words, the difference of potential between C and A, due to the battery E, is equal to the electromotive force of e.

If B be the resistance of E, R the resistance of the rest of the main circuit, and r the resistance of AC, the current along AD is $\dfrac{E}{B+R}$; and the difference of the potentials at AC is $\dfrac{Er}{B+R}$, which is equal to e.

Hence $$Er = (B+R)e.$$

The value of R is then increased by some quantity, which we may call x, and a new position found for C. Suppose the increase in the value of AC to be y. Then we have

$$E(r+y) = (B+R+x)e.$$

Eliminating E, e, between these two equations, we find

$$B = \frac{x}{y} r - R.$$

A difficulty that occurs with this method is due to changes in the value of the electromotive force with the current passing through the battery, since it is assumed that E retains the same value when the conditions of the circuit are altered. It seems, however, that the change only becomes appreciable some little time after the starting of the current, and for most practical purposes the difficulty may be avoided by keeping the battery circuit open, and closing it only an instant before the observation of the

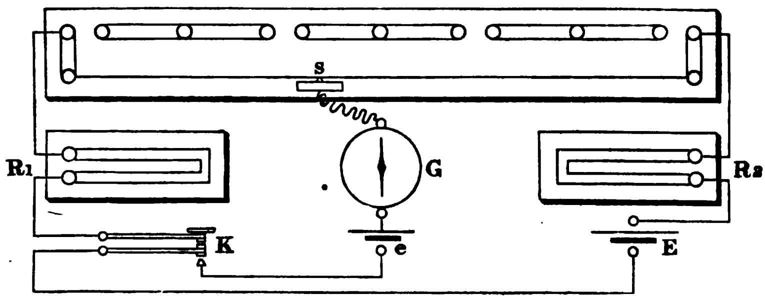


FIG. 57.

galvanometer is made. This is effected with a particular form of key, in which one motion of the hand closes first the battery circuit, and immediately afterwards the connexion at C; and Beetz' method is always described as involving the use of this.

128.] Fig. 57 shows the connexions of the apparatus to a metre bridge. E is the battery whose resistance is to be determined, e the battery of smaller electromotive force used for reference, G the galvanometer, K the special key used for this test. The main circuit of E when K is closed consists of the two resistance boxes R_1, R_2, and the slide wire of the metre bridge. The resistance between the points to

which the terminals of the circuit of e are connected—a resistance called r in Article 127—is the box R_1, together with the portion of the slide wire AS. When the key K is pressed the main circuit of E is first completed, and immediately afterwards the circuit of e. The position of the slider S′ has to be adjusted till no deflection is observed on G when the key is closed. If this position cannot be found on the whole length of the scale, the resistances open in R_1, R_2, must be changed.

128 a.] It may be found that on pressing the key the galvanometer needle first gives a sharp movement, and afterwards a slowly increasing deflection. The first of these—the sharp movement—is the indication looked for, that the potentials at the points to which the ends of the galvanometer circuit are connected do not differ exactly by e. The slowly increasing deflection that follows indicates that the electromotive force of the battery is changing with the passage of the current through it—a change known as polarization. Since the battery must be in the same condition for both parts of the experiment, this gradually increasing deflection is to be neglected, and the adjustment of the resistances to be considered perfect when the needle starts off on this steady movement without any initial jerk.

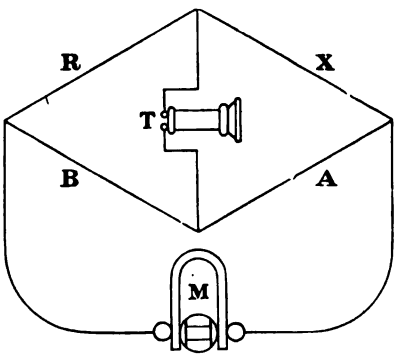


FIG. 58.

129.] (*c*) The third method employed for the measurement of the resistance of batteries is due to Kohlrausch, and is equally applicable to all electrolytic cells, whether they are independent sources of electromotive force or not. It consists in connecting the cell, or cells, in one arm of a Wheat-

stone's bridge, and replacing the battery with a source of rapidly alternating or intermittent currents, such as a small magneto machine, or a battery provided with an interrupter; a telephone is used instead of a galvanometer. The arrangement is shown in Fig. 58. The resistances of A, B, R are adjusted until the peculiar noise due to the interrupter disappears — perfect silence from all sound is practically unattainable—and then if the magnetic inductance and electrostatic capacity of all the conductors are negligible, we have $X = \frac{A}{B} \cdot R$. During this test a steady current is passing from the battery through the other conductors. This has,

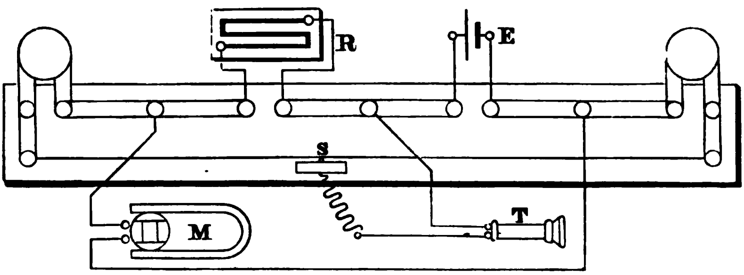


Fig. 59.

of course, no effect on the telephone, which is only affected by variations in the current passing through it, and the noise in which is due to the magneto or interrupter. The capacity and inductance of the coils of a well-made resistance box are so small that they can be entirely neglected in practical measurements. For very accurate work they may perhaps require to be considered.

Fig. 59 shows the connexions of the apparatus for this test with a metre bridge. Here E is the battery to be measured, M the magneto generator, and T the telephone receiver. The resistance box R, and slide S, are to be adjusted till no noise is heard in the telephone due to the magneto.

130.] Several other methods have been proposed. Some,

among which are Ohm's and Lord Kelvin's, depend on observing the change in the current given by the battery when the resistance of the circuit is altered. Another, the principle of which has been described in the chapter on the metre bridge, consists in measuring the resistance of the battery with Wheatstone's bridge, treating it as that of a metallic conductor; two measurements are made with the testing battery connected in opposite senses, and the mean of the results taken for the real value of the resistance. This is the common practice in testing the resistance of submarine cables, in which a difference of potential may, and generally does, exist between the earth plates at the near and distant ends of the cable. It is clear that a difference of potential between the two earth plates, due to some action in the earth's mass, has an effect equivalent to that of connecting AB with a conductor which includes a battery. The testing battery used is generally of considerable electromotive force, a Leclanché battery of 18 or 20 cells being common; and several measurements are made in succession, the battery being reversed after each test. The mean of the results is the resistance of the cable.

131.] All the methods that have been described depend on observing the effect of making some alteration in the battery circuit and the current flowing through it. Thus in Mance's method the current passing from the battery through the system of conductors is changed repeatedly by opening and closing a short circuit key; in Beetz' method successive observations are made of the resistance intercepted between two points, whose potentials differ by the same amount, when the current flowing through the circuit is altered; in Kohlrausch's method the current flowing in the system of conductors is alternately and rapidly increased and diminished by that supplied by the magneto machine.

The theory of all these rests on an assumption that the values of the electromotive force, and the resistance of

a battery, are independent of the current passing through it; and the assumption that E and R are constants is the basis of the mathematical statement of Mance's method given in Appendix V, and of Beetz' method on page 156. The value of E, however, is known not to be constant, but diminishes when the current passing is increased.

132.] We may examine, however, more closely the application of Kohlrausch's method. The theory of Wheatstone's bridge shows that if X, the resistance to be measured, is merely that of a metallic conductor, and if $BX = AR$, no change in the electromotive force at M will affect the circuit of T: and if a steady current be flowing in T, due to a constant electromotive force placed in one of the arms, say in X, this current will not be altered by changes of electromotive force or of resistance at M. If, however, the electromotive force in X is not constant, but varies with the current passing through it, then variations in M will, by altering the currents in the system, affect the electromotive force in X, and consequently the current flowing in T due to X. In other words, even though $BX = AR$, an alternate current machine at M will produce a noise in the telephone if its alternations make corresponding changes in the battery at X. Thus far then the method is imperfect.

It has already, however, been pointed out that some little time seems to be required for the polarization of a battery to become appreciable, and Kohlrausch took advantage of this property to prevent the current from M causing any polarization of the battery in X by making its alternations very rapid. The telephone enabled him to observe the alternating current, which could not have been done with a galvanometer. The electromotive force of the battery may be slowly changing since a current is passing from it through the conductors of the system: this, however, is not heard in the telephone, which is only sensitive to sudden variations, and it has nothing to do with the resistance of the battery.

The value of Kohlrausch's method lies then in the fact that the changes in the testing current are made and reversed so rapidly, that no work is done by it in polarizing the battery, and this device succeeds in cases where the time required to take the observation in Mance's or Beetz' methods is long enough to allow serious polarization to occur.

The application of methods, other than Kohlrausch's, is limited by considerations of the speed with which the cell polarized, and the circumstances of practical cases may be summarized roughly as follows:—

133.] (*a*) The electromotive force available on a closed circuit from two dissimilar conductors placed in a simple electrolyte is less than that upon open circuit, and is less when a large current is passing than with a small current. This is caused by the accumulation of the products of electrolysis upon the electrodes, producing a new electromotive force in opposition to that of the battery. This polarization appears very rapidly on closing the circuit, and continues to increase for a considerable time.

134.] (*b*) The electromotive force available from two dissimilar conductors placed in an electrolyte provided with a depolarizer is probably always less on closed circuit than an open circuit; but the difference may be very small, and depends on the activity of the depolarizer. In Grove's and Bunsen's cells, or in a bichromate cell, the action of the strong nitric or chromic acid is so rapid that gaseous products of electrolysis do not appear, and the electromotive force is appreciably the same when a large current is passing, and when it is on open circuit.

The peroxide of manganese in a Leclanché cell is comparatively inert, and if anything more than a very small current be taken from the cell, hydrogen is deposited on the carbon plate more rapidly than the manganese can oxydize it. The effective electromotive force of the cell

is consequently diminished. The value of a Leclanché cell depends of course on its singular inactivity when on open circuit, so that it can be left for a long time unattended without deterioration. As a continuous source of electricity it is not effective.

135.] (*c*) The resistance of an electrolytic solution independently of any plates or electrodes is independent of the current passing, and the specific resistance of a solution of given strength, and at a fixed temperature, is probably as characteristic a constant as that of a metal.

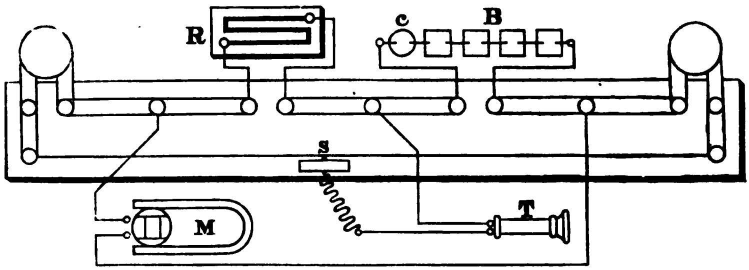


Fig. 60.

It may be measured by comparing the potentials between platinum wires fused through the side of a glass tube holding the solution and conveying the current. The difference of potential is found to be proportional to the current flowing through the solution. No current must be taken from the platinum wires, or the measurement becomes complicated by electrolytic action at those points.

136.] (*d*) The resistance between two electrodes in an electrolytic solution, whether containing a depolarizer or not, is independent of the current passing. This may be shown by testing it in series with a battery by Kohlrausch's method, as follows. The electrolytic cell C, which may conveniently be a U tube, is joined in series with a battery B. composed of an even number of cells of the same kind, shown in Fig. 60 as four. It is connected to a metre bridge

with a resistance box, telephone, and an alternating magneto generator. The resistance box R and the slide S are adjusted till the telephone is as nearly silent as possible. This determines the joint resistance of B and C when the current from B is flowing.

Half of the cells of B are then reversed in direction, so that their electromotive force is opposed to that of the other half, and the current flowing is zero, or at least very small, and S is again adjusted. The author was unable in his experiments to find any difference in the positions of S in the two arrangements to obtain silence in the telephone. C was a small U tube containing sulphate of zinc, and various electrodes were tried of brass, copper, and zinc. The polarization of the cell was so rapid with a direct current that no consistent series of measurements could be obtained with either Beetz' or Mance's methods. The delicacy of the arrangement was such that the value of the resistance could be determined within about $\frac{1}{2}$ per cent. The condition which limits the accuracy of measurements made with this method lies in the difficulty of deciding exactly when the minimum noise is heard in the telephone.

137.] This conclusion that the resistance of a cell is a constant for all currents only holds, of course, so long as the condition of the cell is unchanged. If, by the operations of testing, the strength of the solution be altered, or the surfaces of the electrodes changed, the resistance will also be modified.

138.] If we may regard the transference of electricity in an electrolyte purely as a phenomenon of convection, and the function of the electrodes only to attract and discharge the free wandering ions, it seems not unreasonable to suppose that the heat developed in the cell is only that produced by the collisions of the two opposite processions. Since the number of collisions is proportional to the square of the number of members of each procession, the heat developed will also be

proportional to the square of the current, and the resistance of the cell will be independent of the current.

139.] The practical conclusion of this discussion of the merits of different methods is that Kohlrausch's method is to be preferred for measuring the resistance of electrolytic cells, since it is quite independent of the electromotive force of the cell, and appears to be universally applicable to cases where there is no electrostatic capacity, or magnetic inductance.

The alternating magneto and a pair of telephones—one for each ear—can be obtained from the telephone manufacturers at so low a price that the cost of the apparatus beyond the Wheatstone's bridge, to be found in every testing room, should not be a drawback to their use.

Small Leclanché cells, or others, which it is desired to measure without taking any direct current from them, may be conveniently connected in pairs so that the electromotive forces oppose one another.

Any of the methods may be used fairly satisfactorily in the case of batteries with active depolarizers; but if any change takes place in the electromotive force of the cell between two parts of the test, the value obtained for the resistance will be in error.

APPENDIX I.

THE MATHEMATICAL THEORY OF WHEATSTONE'S BRIDGE.

WHEATSTONE'S bridge consists essentially of six conductors connecting four points A, B, C, O. An electromotive force E is made to act between two of the points by means of a voltaic battery introduced between B and C. The current between the two other points O and A is measured by a galvanometer Under

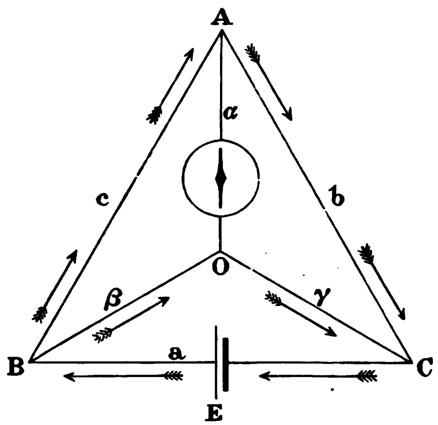


FIG. 61.

certain circumstances this current becomes zero. The conductors BC and OA are then said to be conjugate to each other, which implies a certain relationship between the resistances of the other four conductors, and this relation is made use of in measuring resistances. If the current in OA is zero, the potential at O must be equal to that at A. Now, when we know the potentials

B and C, we can determine those at O and A, provided there is no current in OA, by the rule:

$$A = \frac{Bc + Cb}{b + c}, \qquad O = \frac{B\beta + C\gamma}{\beta + \gamma}.$$

Whence the condition is $b\beta = c\gamma$, where $b\beta$, $c\gamma$ are the conductivities of the respective conductors, and A, B, C, O are the potentials at those points. For convenience, let us make the potential at O zero.

To determine the degree of accuracy of this method we must ascertain the strength of the current in OA[1]. Let a be the conductivity of the battery, and α of the conductor OA; and let E be the E.M.F. of the battery. The arrows show the directions of the currents.

The current—

along BA is $(B-A)c$, that along CO is $C\gamma$.
" AC is $(A-C)b$, " " BO is $B\beta$.
" CB is $(E+C-B)a$, " " AO is $A\alpha$.

Then the sum of the currents flowing to the point A must equal that of those flowing away from it,

$$\text{i.e.}\quad (B-A)c = A\alpha + (A-C)b,$$

similarly for B,

$$(E+C-B)a = (B-A)c + B\beta,$$

and for C,

$$(A-C)b = (E+C-B)a + C\gamma.$$

The three equations may be written

$$A(\alpha+b+c) - Bc - Cb = 0.$$
$$-Ac + B(a+\beta+c) - Ca = Ea.$$
$$-Ab - Ba + C(a+b+\gamma) = -Ea.$$

Whence we have

$$A\begin{vmatrix} \alpha+b+c & -c & -b \\ -c & a+\beta+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix} = E\begin{vmatrix} -c & -b & 0 \\ a+\beta+c & -a & a \\ -a & a+b+\gamma & -a \end{vmatrix},$$

and expanding the right-hand determinant, $= Ea(c\gamma - b\beta)$.

[1] The statement of the theory of Wheatstone's bridge is thus far extracted from *Electricity and Magnetism*, vol. i, Clerk Maxwell.

Let the current through the conductor AO be called u so that $u = \text{A}a$.

Then

$$u\begin{vmatrix} a+b+c & -c & -b \\ -c & a+\beta+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix} = \text{E}\alpha a\,(c\gamma - b\beta) \ldots \quad (1)$$

From the form of this expression we observe

i. that if $\dfrac{b}{c} = \dfrac{\gamma}{\beta}$, so that $c\gamma - b\beta = 0$, $\text{A} = 0$ and there is no current through the circuit AO.

ii. that $c\gamma - b\beta = 0$ is also the condition that there shall be no current through the circuit BC when a source of electromotive force is placed in AO. Two conductors of a network for which this holds, so that a source of electromotive force placed in either one produces no current in the other, are said to be conjugate to one another.

iii. that giving any general values to the conductors of a network of this form, the current in AO due to an electromotive force in BC is equal to that which would be produced in BC by the same electromotive force placed in AO.

For consider the determinant on the left-hand side of equation (1), viz.

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+\beta+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix}.$$

Add the first and second columns to the third, and of the new determinant thus formed add the first and second rows to the third, neither of which operations changes its value.

It becomes

$$\begin{vmatrix} a+b+c & -c & a \\ -c & a+\beta+c & \beta \\ a & \beta & a+\beta+\gamma \end{vmatrix},$$

which, by obvious changes of sign and transpositions, becomes

$$\begin{vmatrix} a+\beta+c & -c & -\beta \\ -c & a+b+c & -a \\ -\beta & -a & a+\beta+\gamma \end{vmatrix}.$$

This determinant may also be obtained by interchanging α with a, and b with β, in the original determinant, and shows that this pair of interchanges can be made without affecting the value of the expression. The symmetry of the original determinant also shows that the same proposition holds of the other two possible pairs of interchanges, viz. $b\beta$ and $c\gamma$; or $c\gamma$ and αa.

These interchanges have moreover no effect upon the right-hand side of equation (1). Hence they have no effect upon the value of u given by that equation: except that when α has been interchanged with a, u represents the current in the α conductor. The interchange of b with β, and c with γ has no physical significance for our present purpose, but the interchange of αa and $b\beta$, or of αa and $c\gamma$ is equivalent to removing the electromotive force from the α conductor, and placing it

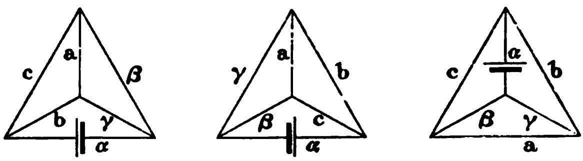


Fig. 62.

in the a conductor. That this is so may be seen from the diagram, Fig. 62, of the three equivalent arrangements in which Fig. 1 is the result of interchanging α with a and b with β; Fig. 2 of α with a and c with γ; and Fig. 3 of moving the electromotive force from the α circuit to the a circuit.

Hence we have proved that the current now passing in the α conductor is the same as previously passed in the a conductor.

iv. The last argument also supplies a rule for determining where to place the electromotive force, and where the galvanometer. The interchange of these two is more than the transfer of the seat of electromotive force just considered. It implies the interchange of the two conductors, in one of which the force was situated, and is equivalent to the interchange of αa without that of either $b\beta$ or $c\gamma$. Since, however, the further interchange of both these has no effect on the expressions, we may assume the change considered to be that of all three.

The reduction in the value of the determinant is then

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & \alpha+\beta+c & -\alpha \\ -b & -\alpha & \alpha+b+\gamma \end{vmatrix} - \begin{vmatrix} \alpha+b+c & -c & -b \\ -c & a+\beta+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix},$$

having interchanged only a and α, which difference is equal to

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & \alpha+\beta+c & -\alpha \\ -b & -\alpha & \alpha+b+\gamma \end{vmatrix} - \begin{vmatrix} \alpha+\beta+\gamma & -\gamma & -\beta \\ -\gamma & a+b+\gamma & a \\ -\beta & -a & a+\beta+c \end{vmatrix},$$

where all three pairs have been interchanged. From the first form it is clear that the expression is zero where $a = \alpha$, i. e. $(a-\alpha)$ is a factor of it. The symmetry of the second form shows that $(\beta-b)$ and $(\gamma-c)$ are also factors. Hence the whole is equal to $M(a-\alpha)(\beta-b)(\gamma-c)$. To determine M, let us write $a = 0$, $b = 0$, $c = 0$: the value of the first of the two determinants is then $\alpha\beta\gamma$, while that of the second is zero. Hence $M = 1$, and the difference becomes generally $(a-\alpha)(\beta-b)(\gamma-c)$.

If this is negative, the value of the original determinant is increased by the change and the value of u is diminished; or in other words, the one of the two arrangements which makes $(a-\alpha)(\beta-b)(\gamma-c)$ negative will give the larger value to u.

This gives us the rule that the three larger conductors, taking one from each pair $a\alpha$, $b\beta$, $c\gamma$, should form a triangle, and the three smaller conductors should be concurrent to give the largest value to u. This rule is commonly given, that of the two resistances—the galvanometer and the battery—the higher should connect the points of junction of the two highest, and of the two lowest of the other four resistances, in order to give the largest deflection on the galvanometer for a given error of balance.

In using this arrangement for measuring resistance, or its inverse conductivity, the place of one of the four conductors b, c, β, γ is occupied by that under examination, and the other three by conductors whose values are known, one or more of which are adjustable. The battery is connected in the circuit BC, and the known resistances are adjusted till a galvanometer placed in AO shows no deflection. The value of the unknown can then be obtained from those of the known conductors from

the expression connecting the conductivities b, c, β, γ, viz. $c\gamma - b\beta = 0$. It will be noticed that if $c\gamma - b\beta = 0$, when b, c, β, γ are the conductivities, the same equation holds when they represent the resistances of the four conductors; but the forms of the expressions in the investigation that follows are simpler when they represent conductivities. We proceed to determine the values of the various conductors which will give the greatest sensitiveness to the apparatus in measuring a given resistance. The accuracy with which this measurement can be made depends on the degree to which small errors in the exact adjustment of their relative values can be observed, and the limit of the accuracy of adjustment is reached when the remaining error produces no appreciable deflection of the galvanometer needle. It is when the adjustment is nearly complete that the greatest sensitiveness to small changes of adjustment is required, and we proceed to calculate what arrangements of conductivities will give the largest deflection of the galvanometer, supposing them nearly to satisfy the condition

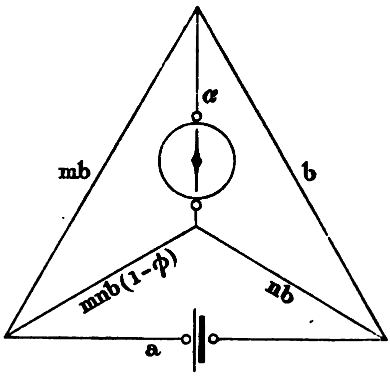


FIG. 63.

$$c\gamma - b\beta = 0.$$

In the diagram (Fig. 63) we will take b to be the conductivity of the conductor under measurement. Let us write $c = mb$, $\gamma = nb$, and $\beta = mnb(1-\phi)$, so that $c\gamma - b\beta = mnb\phi$. Then, since $c\gamma - b\beta$ nearly equals zero, ϕ is a small quantity. Substituting these expressions in equation (1), we have

$$u \begin{vmatrix} a+b(1+m) & -mb & -b \\ -mb & a+mb(1+n) & -a \\ -b & -a & a+b(1+n) \end{vmatrix} = \text{E}\alpha a mnb^2\phi.$$

The part involving ϕ in the middle term of the determinant is negligible compared with the others and is omitted.

Expanding and dividing both sides by b, we have

$$u\{a(1+n)+nb(1+m)\}\{a(1+m)+mb(1+n)\}$$
$$= \mathrm{E}aamn\phi b \quad . \quad . \quad . \quad (2)$$

We observe about this equation that the conductivity of the whole battery circuit, supposing no current to pass through the galvanometer, is $\dfrac{amb(1+n)}{a(1+m)+mb(1+n)}$, and this we may call B. The conductivity of the galvanometer circuit, supposing a source of electromotive force to be placed in it, no current passing through the battery, is $\dfrac{anb(1+m)}{a(1+n)+nb(1+m)}$, which we call Γ. Equation (2) then becomes

$$u(1+m)(1+n)b = \mathrm{E}.\mathrm{B}.\Gamma.\phi$$
$$= \Gamma.\phi.\Upsilon,$$ where Υ is the whole current flowing from the battery.

We remark that

$$(1+m)(1+n)b = b+mb+nb+mnb$$
= the sum of the conductivities of the four sides of the quadrilateral.

Accordingly, equation (2) states that the galvanometer current

i. is proportional to the E. M. F. of the battery E;

ii. is proportional to the conductivity of the battery circuit B;

the product of these two being the whole battery current Υ.

iii. is proportional to the conductivity of the galvanometer circuit Γ;

iv. is proportional to the error of balance ϕ, supposed to be small;

v. is inversely proportional to the sum of the conductivities of the four sides of the quadrilateral.

The quotient of the last two quantities, viz. $\dfrac{\phi}{c_1+c_2+c_3+c_4}$, may be written $\dfrac{r_1 r_4 - r_2 r_3}{r_1+r_2+r_3+r_4}$, $c_1 c_2 c_3 c_4$ and $r_1 r_2 r_3 r_4$ being the conductivities and the resistances respectively of the four sides of the quadrilateral, the balance being supposed to be nearly perfect.

We now proceed to determine the values of m and n which will give a maximum value to u.

We have

$$\mathrm{E}aa\phi b \frac{1}{u} = \left\{a\left(1+\frac{1}{n}\right)+b(1+m)\right\}\left\{a\left(1+\frac{1}{m}\right)+b(1+n)\right\}.$$

So $$\mathrm{E}aa\phi b \frac{d}{dm}\left(\frac{1}{u}\right) = \frac{abn(m^2-1)+(b^2m^2n-aa)(1+n)}{m^2n},$$

and $$\mathrm{E}aa\phi b \frac{d}{dn}\left(\frac{1}{u}\right) = \frac{abm(n^2-1)+(b^2mn^2-aa)(1+m)}{mn^2}.$$

If mn have such values as give u and consequently $\frac{1}{u}$ a critical value $\frac{d}{dm}\left(\frac{1}{u}\right)=0$ and $\frac{d}{dn}\left(\frac{1}{u}\right)=0$.

So $abn(m^2-1)+(b^2m^2n-aa)(1+n)=0$,
and $abm(n^2-1)+(b^2mn^2-aa)(1+m)=0$.

By cross multiplication we get

$$(1+m)(1+n)(aa-b^2m^2n^2)(aa-b^2mn)=0.$$

This equation indicates four different sets of values for mn, two of which if they are all critical give maximum values to u, and two minimum. In the first two of these either $m=-1$ or $n=-1$, and since we can assign no physical meaning to a negative conductivity, these give no solution of the problem. Equating the last factor to zero, viz. $aa-b^2mn=0$, and substituting in the two preceding equations, we obtain

$$m = -\frac{a(a+b)}{b(a+b)} \text{ and } n = -\frac{a(a+b)}{b(a+b)},$$

which being negative also have no meaning.

The remaining factor gives us the equation

$$aa-b^2m^2n^2=0.$$

Substituting this in the previous equations we have

$$m = \left\{\frac{a(a+b)}{b(a+b)}\right\}^{\frac{1}{2}} \text{ and } n = \left\{\frac{a(a+b)}{b(a+b)}\right\}^{\frac{1}{2}},$$

the positive sign being taken in both cases.

The second differentials of $\frac{1}{u}$ with regard to m and n are as follows:

$$\text{E}\alpha a\phi b \frac{d^2}{dm^2}\left(\frac{1}{u}\right) = \frac{2\alpha}{m^3}\left\{b + a\left(1 + \frac{1}{n}\right)\right\}.$$

$$\text{E}\alpha a\phi b \frac{d^2}{dmdn}\left(\frac{1}{u}\right) = b^2 + \frac{\alpha a}{m^2n^2} = \frac{2\alpha a}{m^2n^2} \text{ since } b^2m^2n^2 = \alpha a.$$

$$\text{E}\alpha a\phi b \frac{d^2}{dn^2}\left(\frac{1}{u}\right) = \frac{2a}{n^3}\left\{b + \alpha\left(1 + \frac{1}{m}\right)\right\}$$

$$\frac{d^2}{dm^2}\left(\frac{1}{u}\right) \text{ and } \frac{d^2}{dn^2}\left(\frac{1}{u}\right) \text{ are of the same sign,}$$

and $\quad 4\frac{d^2}{dm^2}\left(\frac{1}{u}\right) \times \frac{d^2}{dn^2}\left(\frac{1}{u}\right) - \left\{\frac{d^2}{dmdn}\left(\frac{1}{u}\right)\right\}^2$ is positive.

Hence the value of $\left(\frac{1}{u}\right)$ is critical. For positive values of mn all these second differentials are of the same sign as ϕ, and consequently of the same sign as u. Hence positive values of mn obtained by solving $\frac{d}{dm}\left(\frac{1}{u}\right) = 0$ and $\frac{d}{dn}\left(\frac{1}{u}\right) = 0$ make the value of $\frac{1}{u}$ a minimum or a maximum as u is positive or negative, and u itself a maximum when it is positive and a minimum when it is negative: that is they make its value an arithmetical maximum.

Again, these values of m and n make $mnb = \sqrt{\alpha a}$, so that in Wheatstone's bridge, the conductivity of the bridge coil farthest from the conductor under measurement should be the geometric mean of those of the battery and galvanometer.

Substituting these values in equation (2) we have

$$u\{\sqrt{\alpha b} + \sqrt{ab} + \sqrt{(\alpha + b)(a + b)}\}^2 = \text{E}\alpha a\phi b. \quad . \quad (3)$$

We proceed to determine what will be the best value of a for measuring any given conductivity b, that is, to what resistance should the galvanometer be wound, so as to give the largest deflection with a small error of balance ϕ in measuring a given resistance. Now, if we wind a galvanometer of given form and dimensions with various sizes of wire, so that its resistance takes different values, we find that, if ϵ be the E. M. F. applied to the terminals, k its conductivity, and v the current, so that $v = \epsilon k$,

the action on the galvanometer needle is proportional to $\epsilon k^{\frac{1}{2}}$, i.e. to $vk^{-\frac{1}{2}}$. Hence, in the equation we are discussing, we have to determine from equation (3) what value of a will make $ua^{-\frac{1}{2}}$ a maximum.

Now $ua^{-\frac{1}{2}} = \text{E}aa^{\frac{1}{2}}\phi\{\sqrt{ab}+\sqrt{ab}+\sqrt{(a+b)(a+b)}\}^{-2}$.

We write $$\frac{d}{da}(u.a^{-\frac{1}{2}}) = 0,$$

i.e. $$\frac{d}{da}\{a^{\frac{1}{2}}(\sqrt{ab}+\sqrt{ab}+\sqrt{(a+b)(a+b)})^{-2}\} = 0.$$

This gives, on differentiating,

$$(\sqrt{a}-\sqrt{a})\sqrt{b}\sqrt{a+b} = (a-b)\sqrt{a+b}.$$

This may be written in the form

$$\frac{\sqrt{a}}{\sqrt{a+b}}\cdot\frac{\sqrt{b}}{\sqrt{a+b}} - \frac{\sqrt{a}}{\sqrt{a+b}}\cdot\frac{\sqrt{b}}{\sqrt{a+b}} = \frac{a-b}{a+b}. \quad . \quad . \quad (4)$$

Writing $$-\frac{\sqrt{a}}{\sqrt{a+b}} = \cos\theta, \quad \frac{\sqrt{b}}{\sqrt{a+b}} = \sin\theta,$$

and $$-\frac{\sqrt{a}}{\sqrt{a+b}} = \sin\psi, \quad \frac{\sqrt{b}}{\sqrt{a+b}} = \cos\psi,$$

equation (4) becomes $\cos(\theta+\psi) = \cos 2\theta$,

$$\text{i.e.} \quad \pm 2\theta = 2n\pi+\theta+\psi.$$

This gives four sets of values of the trigonometrical functions of θ in terms of ψ, viz. when $\theta=\psi$, and when $3\theta = -(2n\pi+\psi)$, i.e. when $\theta = -\frac{\psi}{3}, \ -\frac{2\pi+\psi}{3}, \ -\frac{4\pi+\psi}{3}$.

Now, since $\sqrt{a}$, $\sqrt{a}$, $\sqrt{b}$ are all necessarily positive from the physical considerations, θ must lie in the second quadrant, and ψ in the fourth quadrant. Hence θ cannot equal ψ, and the first solution is inadmissible.

Again,

$-\frac{\psi}{3}$ lies in the 3rd quadrant, so this value of θ is inadmissible,

$-\frac{4\pi+\psi}{3}$ „ 1st „ „ „

$-\frac{2\pi+\psi}{3}$ lies in 2nd quadrant, so this is the only possible solution.

This gives us $\sqrt{a} = \sqrt{b}\tan\left(\frac{\pi}{6} + \frac{1}{3}\tan^{-1}\frac{\sqrt{a}}{\sqrt{b}}\right)$,

which is the only value of a giving a critical value to $ua^{-\frac{1}{2}}$. To determine whether this is a maximum or a minimum, we proceed to differentiate again, writing v for

$$\sqrt{ab} + \sqrt{ab} + \sqrt{(a+b)(a+b)},$$

and writing $$U = a^{\frac{1}{2}}v^{-2}.$$

We find for values of a, which make $\frac{dU}{da}$ zero,

$$\frac{d^2U}{da^2} = a^{-\frac{3}{2}}v^{-2}\left\{-\frac{1}{8} - \frac{ba}{2(a+b)}v^{-1}\frac{\sqrt{a+b}}{\sqrt{a+b}}\right\};$$

now these radicals were all introduced as positive quantities; hence $\frac{d^2U}{da^2}$ is negative, and the value given to $ua^{-\frac{1}{2}}$ is an arithmetical maximum.

The sensitiveness of the apparatus will always be increased by reducing the resistance of the battery.

The sum of these results is as follows.

Where b is the conductivity of the conductor under examination, and a that of the battery to be employed, the best conductivity for the galvanometer is a, where

$$a = b\left\{\tan\left(\frac{\pi}{6} + \frac{1}{3}\tan^{-1}\sqrt{\frac{a}{b}}\right)\right\}^2.$$

The conductivities of the four conductors of the quadrilateral are to be

$$b,\ b\left\{\frac{a(a+b)}{b(a+b)}\right\}^{\frac{1}{2}},\ b\left\{\frac{a(a+b)}{b(a+b)}\right\}^{\frac{1}{2}},\ \text{and}\ \sqrt{aa}.$$

Of these three the last is placed opposite to the conductor under measurement, and the galvanometer is connected between the points of junction of the first two, and the last two.

As an example, the best resistance to give to a galvanometer for measuring a resistance of 1000 ohms with a battery whose

resistance is 10 ohms would be 400 ohms; and the other arms of the quadrilateral will be respectively 500 ohms, 128 ohms, and 64 ohms, all taken in round figures; the 64 ohms to form the side opposite to the 1000 ohms, and the galvanometer to be connected between the junction of the 1000 ohms and 500 ohms, and that of the 128 ohms and 64 ohms.

APPENDIX II.

Lord Kelvin's Modification of Wheatstone's Bridge for Low Resistances.

Lord Kelvin's method of comparing conductors of low resistance is shown diagrammatically in Figure 64. BQ, RC are the conductors to be compared, and the comparison is made of the conductivities β, γ, between points BQ and CR. A current

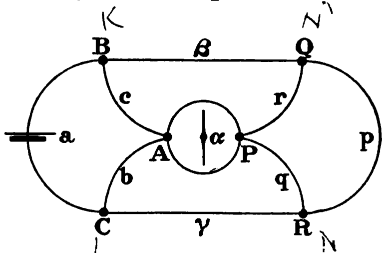


Fig. 64.

is passed from a battery through the two conductors in series, the ends QR being connected by a stout conductor. The terminals of a galvanometer AP are connected by wires to points B, Q, R, C, whose conductivities are c, r, q, b. Two conditions can be determined between β, γ, b, c, q, r under which no current will pass through the galvanometer, and when this is observed a comparison can be made between their values.

The symmetry of the arrangement can be seen from the equivalent diagram in Fig. 65. In referring to this we shall

use the large letters to represent the potentials of the points against which they are placed, while the small letters are the conductivities of the different conductors.

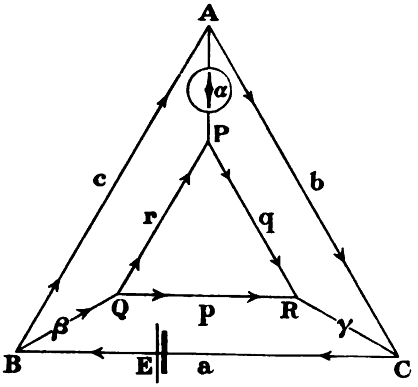


FIG. 65.

From Kirchhoff's law we have

$$\begin{array}{lllll}
\text{at A} & (B-A)c & -(A-C)b- & (A-P)a & =0, \\
\text{at B} & (C-B+E)a & -(B-A)c- & (B-Q)\beta & =0, \\
\text{at C} & (A-C)b & +(R-C)\gamma- & (C-B+E)a & =0, \\
\text{at P} & (Q-P)r & +(A-P)a- & (P-R)q & =0, \\
\text{at Q} & (B-Q)\beta & -(Q-P)r- & (Q-R)p & =0, \\
\text{at R} & (Q-R)p & +(P-R)q- & (R-C)\gamma & =0.
\end{array}$$

Since any one of these six equations can be obtained by adding the others together, only five of them are independent. Also we may give an arbitrary value to any one of the six potentials. Let us then omit the first equation, and write $P=0$. The other five equations become

$$\begin{array}{lllll}
Ac-B(a+\beta+c) & +Ca & & +Q\beta+Ea & =0, \\
Ab+Ba & & -C(a+b+\gamma) & +R\gamma-Ea & =0, \\
Aa+Qr & & +Rq & & =0, \\
Bb-Q(p+\beta+r)+Rp & & & & =0, \\
C\gamma+Qp & & -R(p+q+\gamma) & & =0
\end{array}$$

From which we have

$$A\begin{vmatrix} c-(a+\beta+c) & a & \beta & 0 \\ b & a & -(a+b+\gamma) & 0 & \gamma \\ a & 0 & 0 & r & q \\ 0 & \beta & 0 & -(p+\beta+r) & p \\ 0 & 0 & \gamma & p & -(p+q+\gamma) \end{vmatrix}$$

$$+E\begin{vmatrix} a-(a+\beta+c) & a & \beta & 0 \\ -a & a & -(a+b+\gamma) & 0 & \gamma \\ 0 & 0 & 0 & r & q \\ 0 & \beta & 0 & -(p+\beta+r) & p \\ 0 & 0 & \gamma & p & -(p+q+\gamma) \end{vmatrix} = 0.$$

The condition that there shall be no current through the galvanometer is that $A = 0$, i.e. that the coefficient of E in the last equation is zero.

Expanding the right-hand determinant we obtain

$$(b\beta - c\gamma)(qr + rp + pq) + \beta\gamma(br - cq).$$

This expression equals zero if both

$$b\beta - c\gamma = 0 \text{ and } br - cq = 0,$$

i.e. if

$$\frac{b}{c} = \frac{\gamma}{\beta} = \frac{q}{r}.$$

If we make $br - cq = 0$ by construction, and then adjust either β or γ till there is no galvanometer deflection we know the ratio of β to γ. Also if p be very large compared with r and q, β and γ, i.e. if the connection PQ between the conductors be very good, the exact equality of $\frac{b}{c}$ and $\frac{q}{r}$ becomes of minor importance. If p is infinitely large and the connection is perfect the condition is reduced to $\frac{b}{c} = \frac{\gamma}{\beta}$ and the whole arrangement becomes equivalent to the ordinary form of Wheatstone's bridge.

To determine the best dimensions to be given to the various parts, we will take a case when balance has already been nearly attained, as was done in the Wheatstone's bridge. Let us assume the conductors under comparison to be mK and $K-\phi$, ϕ being a very small quantity, so that the ratio between them is m. The other conductors we take to be $k.k$, $mk.mk$, and the

galvanometer conductivity to be G. The arrangement is shown in Fig. 66. When the current through the galvanometer is very small, the currents along A, B, C, D are clearly equal. Let this current be C.

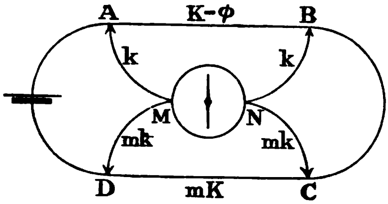


FIG. 66.

Then, if the potentials at B, C are β, γ, the potential at A is $\frac{C}{K-\phi}+\beta$, which we write $\frac{C}{K}+x+\beta$, x being a small quantity. The potential at D is $-\frac{C}{mK}+\gamma$.

Then, if μ, ν be the potentials at M, N, we have, by Kirchoff's law, at M,

$$\left(\frac{C}{K}+x+\beta-\mu\right)k-(\mu-\nu)G-\left(\mu+\frac{C}{mK}-\gamma\right)mk=0,$$

and at N, $$(\beta-\nu)k+(\mu-\nu)G-(\nu-\gamma)mk=0.$$

Subtracting, we have

$$(\mu-\nu)(2G+k+mk)=kx.$$

Observing that for a given difference of potential at the terminals the deflecting force on the needle of any given galvanometer is approximately proportional to the square root of the conductivity, we have in this case that the deflecting force is proportional to $\frac{kxG^{\frac{1}{2}}}{2G+k(1+m)}$.

Of the values of G and k, which will make this a maximum, we observe that k should be infinite. This, however, is impracticable in view of the use of the instrument, which requires k and mk to be so small that the contact resistances at A, B, C, D are negligible in comparison. The value of k must be determined then from practical considerations, and m is a fixed quantity. To determine G write $G=a^2k$; substituting in the expression

for the deflecting force and differentiating with regard to k, we obtain a maximum value when

$$\phi^2 = \frac{1+m}{2},$$

this gives
$$G = \frac{1+m}{2} k,$$

and the conductivity of the galvanometer is a mean between those of the conductors MA, MB. This gives us that

$$\mu - \nu = \frac{x}{2(1+m)},$$

and the galvanometer current is $\dfrac{kx}{4}$.

Now x is the difference of potential between the two ends of the small length of scale wire by which one of the sliders is out of adjustment, and is equal consequently to the product of the resistance of that length, multiplied by the current along it. If that resistance be P', we have

$$x = P' \times C.$$

We observe then that the galvanometer current, which

$$= \frac{k}{4} \cdot P' \times C,$$

i. is proportional to the length of scale wire by which the slider is out of exact adjustment, and is independent of the total length of scale between the sliders, i.e. of the value of K;

ii. is proportional to the testing current along the slide wire;

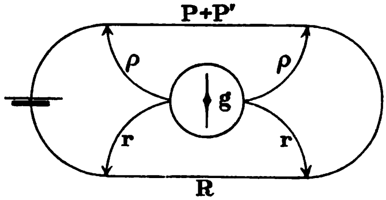


FIG. 67.

iii. is not independent of the value of m, since the galvanometer resistance assumed in obtaining this result is a function of m.

Putting the results we have obtained in terms of resistances, let P + P′. R be those of the main conductors, $r\rho$ of the potential conductors, g of the galvanometer, and $\frac{P}{R} = \frac{\rho}{r}$. The arrangement is shown in Fig. 67.

The electromotive force acting on the galvanometer is generally

$$\frac{rg}{2r\rho + g(r+\rho)} \cdot P' \cdot C;$$

the best value for g is $$\frac{2r\rho}{\rho + r},$$

which value of g makes the electromotive force

$$\frac{r}{2(r+\rho)} \cdot P' \cdot C.$$

APPENDIX III.

Electromotive Forces of Contact at the Junctions of Metre Bridges.

For the discussion of the electromotive forces of contact at the junctions of a metre bridge consider the instrument shown diagrammatically in Fig. 68.

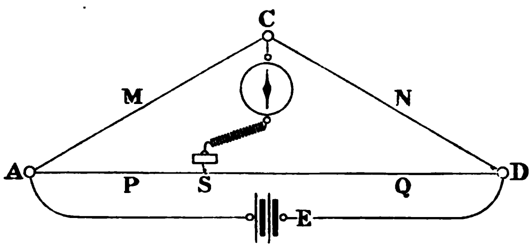


Fig. 68.

In this AD is the bridge, consisting in whole or in part of the slide wire. MN are the resistances placed in the arms AC, CD: and PQ the resistances into which AD is divided by the slider. Then the electromotive force in the galvanometer circuit consists of two parts: one is that due to the battery whose E. M. F. is E, and is equal to $\frac{E}{B} \cdot \frac{QM - PN}{P + Q + M + N}$ supposing $QM - PN$ to be small, and where B is the resistance of the whole battery circuit consisting of the resistance AED with that of the two parallel conductors ACD and ASD; the other part which we may call ϵ is due to the electromotive forces of contact which exist at all the junctions. These latter, if the whole apparatus is at one temperature, will balance one another, and ϵ will equal

zero. But if there are any differences of temperature between different junctions, and in particular if the junction of the slider and the slide wire be warmed by the neighbourhood of the hand, the electromotive forces will no longer balance, and an effective resultant may appear in the galvanometer circuit. This is the quantity we call ϵ. By the galvanometer circuit is meant the circuit composed of the galvanometer CS in series with the two parallel conductors CAS and CDS. Now such a thermoelectromotive force may appear in all or any of the six arms of the bridge. If it appears in AED, it will merely act in addition to, or opposition to, the testing battery, and will not affect the position of the slider for a balance. If it appears in SC it will act as an E. M. F. in the galvanometer circuit and must be taken into account. If it appears in one of the sides of the quadrilateral ACDS, say in AC, part of the current it produces will pass through the galvanometer CS, while the rest passes along CDSA and CDEA. ϵ is then the sum of the electromotive force in CS, and of some fractions of those in AC, CD, DS, and SA. The total electromotive force acting in the galvanometer circuit is $\epsilon + \frac{E}{B} \cdot \frac{QM - PN}{P+Q+M+N}$. As a result of the existence of ϵ the condition that there shall be no current through the galvanometer is no longer that $QM - PN = 0$ but that

$$\epsilon + \frac{E}{B} \cdot \frac{Q_1 M - P_1 N}{P+Q+M+N} = 0.$$

The position of the slider which shall give values of PQ such that $QM - PN = 0$ is found by reversing the testing battery, readjusting the slider, and taking the mean of the two positions as the one required. Thus on reversing the battery we have

$$\epsilon - \frac{E}{B} \cdot \frac{Q_2 M - P_2 N}{P+Q+M+N} = 0,$$

since $P + Q = P_1 + Q_1 = P_2 + Q_2$.

Subtracting the last equation from the one before it we have

$$(Q_1 + Q_2) M - (P_1 + P_2) N = 0,$$

i. e.

$$\frac{M}{N} = \frac{P_1 + P_2}{Q_1 + Q_2} = \frac{P}{Q}.$$

Mr. R. T. Glazebrook has discussed the determination of ϵ by making two determinations of the value of $\frac{P}{Q}$ with largely different values of B, i.e. by inserting an additional resistance, which we may call ρ, in the battery circuit.

We have $$\epsilon + \frac{E}{B} \cdot \frac{Q_1 M - P_1 N}{P+Q+M+N} = 0.$$

Increasing B by ρ we have

$$\epsilon + \frac{E}{B+\rho} \cdot \frac{Q_2 M - P_2 N}{P+Q+M+N} = 0.$$

Eliminating B we have

$$\epsilon\rho\,(P+Q+M+N) = E\,\{(P_2 - P_1)\,N - (Q_2 - Q_1)\,M\}$$

$= E\,(M+N)\,d$, where d is the resistance of the length of the slide wire between the two positions of the slider. From this equation ϵ may be determined.

APPENDIX IV.

On the Discharge of a Charged Condenser through a Conductor possessing High Resistance.

Suppose that a condenser, whose capacity measured in C.G.S. units is k, is charged from a battery, whose electromotive force is V, receiving thereby a charge kV. Its terminals are then connected by a conductor whose conductivity, also measured in C. G. S. units, is c, so that the opposite charges on the two plates of the condenser recombine through the conductor. Let v be the difference of potential between the condenser plates t seconds after the disconnection of the battery, and the connection of the conductor, kv being then the electrostatic charge.

The electricity lost by either plate of the condenser in any interval of time during the experiment, and conveyed to the other plate, constitutes a current through the conductor, the value of which is known in terms of the conductivity and the difference of the potentials of the plates, and from the statement of this may be obtained the law of the variation of potential. The charge lost by the condenser during the interval between the times t and $t+dt$, dt being so small that the change of v in its duration is inappreciable, is $-k\frac{dv}{dt}\,.\,dt$, and the electricity conveyed by the current during that time is $c\,.\,v\,.\,dt$.

Putting these quantities equal to one another,

$$k\frac{dv}{dt} + cv = 0.$$

Integrating and remembering that when $t = 0$, $v = V$, we have

$$k \log \frac{V}{v} = ct. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Suppose the experiment to be repeated with another conductor whose conductivity is $C + c$, and suppose that in this case the difference of potential between the plates t seconds after disconnection from the battery is v'.

Then
$$k \log \frac{V}{v'} = (C + c)\, t.$$

Subtracting (1) from this result we have

$$k \log \frac{v}{v'} = Ct. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If c be the conductivity of the imperfectly insulating materials, and supports of the condenser, and C that of an external resistance, $C + c$ is the whole conductivity between the condenser terminals when the resistance is connected, c when it is disconnected.

Then equation (2) gives us a relation between k and C from which c is eliminated. This relation may be used to determine the conductivity of an external resistance from the known capacity of a condenser, or inversely, if the conductivity is known, the capacity of the condenser may be ascertained: in neither case need the imperfect insulation of the condenser be taken into consideration.

Writing in (2) r for the resistance of the conductor measured in C. G. S. units so that $Cr = 1$, we obtain

$$kr \log \frac{v}{v'} = t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the capacity of the condenser measured in microfarads be K so that $K = k\, 10^{-15}$, and the resistance of the conductor measured in megohms be R so that $R = r\, 10^{15}$, then $KR = kr$.

If Dd be the deflection observed on the scale of a ballistic galvanometer when the charges of electricity kv, kv' are passed through it we have $\dfrac{D}{d} = \dfrac{v}{v'}$.

Hence $KR \log \dfrac{D}{d} = t$, the equation quoted in Article 121.

Equation (1) may be similarly transformed into the result quoted in Article 123.

In this investigation k has been assumed to be independent of V and of t, which is not strictly the case with condensers having solid dielectrics. The absorption of these is a small part of the total capacity, and may be neglected in operations of measurement, such as these above described, which do not admit of great accuracy.

Again, consider the case of a condenser, charged from a battery of known electromotive force, and discharged through a shunted galvanometer. Let the capacity of the condenser be k, the electromotive force of the battery V, and let the difference of the potentials of the condenser plates at a time t after the beginning of the discharge be v. Let g be the resistance of the galvanometer, l its inductance, and $\frac{g}{n-1}$ the resistance of the shunt. Suppose that the current through the galvanometer at time t is c, and that through the shunt c'.

Then $$v - l\frac{dc}{dt} = gc. \quad \ldots \ldots \quad (1)$$

$$(n-1)v = gc'. \quad \ldots \ldots \quad (2)$$

$$c + c' = -k\frac{dv}{dt}. \quad \ldots \ldots \quad (3)$$

Also when $t = 0$, $v = \text{V}$, and $c = 0$. Therefore from (1), when $t=0$,

$$l\frac{dc}{dt} = \text{V}.$$

Eliminating v and c' from (1), (2) and (3), we have,

$$gkl\frac{d^2c}{dt^2} + \{(n-1)\,l + g^2k\}\frac{dc}{dt} + ngc = 0.$$

Then $$c = \text{A}e^{\alpha t} + \text{B}e^{\beta t},$$

and $$\frac{dc}{dt} = \text{A}\alpha e^{\alpha t} + \text{B}\beta e^{\beta t},$$

where α, β are the roots of $gklx^2 + \{(n-1)\,l + g^2k\}\,x + ngc = 0$, and are both negative: and A, B are constants to be determined.

Putting $t=0$, we find $A+B=0$,

and $$l(A\alpha+B\beta)=V.$$

Then $$c=\frac{V}{l(\alpha-\beta)}(e^{\alpha t}-e^{\beta t}).$$

To find the total amount of the charge which passes through the galvanometer, the expression for c must be integrated between the limits $t=0$, and $t=\infty$; thus,

$$\int_0^\infty c\,.\,dt=\frac{V}{l(\alpha-\beta)}\left[\frac{1}{\alpha}e^{\alpha t}-\frac{1}{\beta}e^{\beta t}\right]_{t=0}^{t=\infty}$$

$$=\frac{V}{l\alpha\beta}=\frac{kV}{n}.$$

Hence the portions of the charge which pass through the galvanometer and its shunt respectively, are inversely proportional to their resistances, and are the same as if neither circuit possessed inductance.

APPENDIX V.

THE THEORY OF SIR HENRY MANCE'S METHOD FOR MEASURING THE RESISTANCE OF A BATTERY.

SIR HENRY MANCE'S method of determining the resistance of a battery is shown in the arrangement of conductors of Fig. 69,

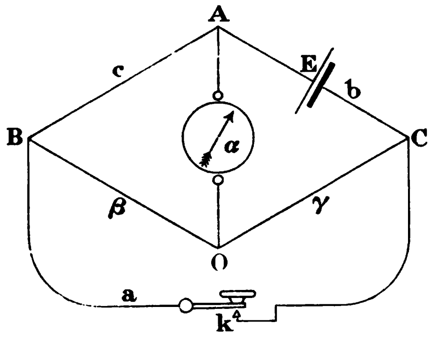


FIG. 69.

similar to that of Wheatstone's bridge, except that the battery is placed in one of the sides of the quadrilateral, and replaced in BC by a tapping key.

a, b, c, α, β, γ are the conductivities of the six conductors, A, B, C, O the potentials at those points, and E the electromotive force of the battery.

We obtain the following equations in the same way as those on page 168:

$$\begin{aligned} \text{A}(\alpha+b+c)-\text{B}c-\text{C}b-\text{E}b &= 0,\\ -\text{A}c+\text{B}(a+\beta+c)-\text{C}a &= 0,\\ -\text{A}b-\text{B}a+\text{C}(a+b+\gamma)+\text{E}b &= 0, \end{aligned}$$

whence we have

$$\text{A}\begin{vmatrix} a+b+c & -c & -b \\ -c & a+\beta+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix} = \text{E}\begin{vmatrix} b & -c & -b \\ 0 & a+\beta+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix},$$

A being the electromotive force at the terminals of the galvanometer.

Expanding, this becomes

$$\text{A}\,\{a\beta\gamma+\beta\gamma(b+c)+\gamma a(c+a)+a\beta(a+b) \\ +(a+\beta+\gamma)(bc+ca+ab)\} \\ = \text{E}b\,\{a(\beta+\gamma)+\gamma(\beta+c)\}.$$

In the particular case when $b\beta = c\gamma$ the last equation may be written as follows, after multiplying both sides by b,

$$\text{A}\{b(a+\beta+\gamma)+a\gamma\}\{bc+ca+ab+b\beta\}=\text{E}b\gamma\{bc+ca+ab+b\beta\}.$$

Dividing both sides by $\{bc+ca+ab+b\beta\}$,

$$\text{A}\,\{b(a+\beta+\gamma)+a\gamma\} = \text{E}b\gamma.$$

The current through the galvanometer, which $= \text{A}a$, is

$$\frac{\text{E}ba\gamma}{b(a+\beta+\gamma)+a\gamma}.$$

This may be written $\text{E}\,\dfrac{b}{b+c}\,.\,\Gamma$, where Γ is the conductivity of the galvanometer circuit, using the word in the sense in which it is used on page 173; i.e. the conductivity of the circuit which starts from A, passes through ABO and ACO in parallel, and back to A through the galvanometer.

Hence the value of A is independent of a, if $b\beta = c\gamma$.

Mance's method of measuring b, the resistance of the galvanometer, consists in adjusting the values of c, β, γ until alterations from 0 to ∞ made in the value of a, by opening and closing the tapping key k, have no effect on the deflection of the galvanometer. The conductivity b of the battery under test is then determined from the relation $b\beta = c\gamma$.

We proceed to determine what resistances should be given to the various conductors to make the method as sensitive as possible.

We assume first that the balance between the conductors

b, c, β, γ is not quite perfect, so that the opening and closing of the key changes the value of A. The values of A in that case are to be determined separately when $a = \infty$ and when $a = 0$; the difference of these two gives the sudden change in the value of A when k is opened or closed. Then values of c, β, γ, a should be arranged to make the effect on the galvanometer needle a maximum.

Suppose that, while $b\beta = c\gamma$, the actual conductivity of BO is $\beta(1+\phi)$, where ϕ is a small quantity, and let A′ be the corresponding value of A. Then

$$A' \begin{vmatrix} a+b+c & -c & -b \\ -c & a+\beta(1+\phi)+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix}$$

$$= E \begin{vmatrix} b & -c & -b \\ 0 & a+\beta(1+\phi)+c & -a \\ -b & -a & a+b+\gamma \end{vmatrix}.$$

Expanding,

$$A'[\{b(a+\beta+\gamma)+a\gamma\}\{bc+ca+ab+b\beta\}$$
$$+ b\beta\phi\{(a+b+c)(a+b+\gamma)-b^2\}]$$
$$= Eb\gamma\{bc+ca+ab+c\phi(a+\gamma)\}.$$

Writing
$$b(a+\beta+\gamma)+a\gamma = P,$$
$$bc+ca+ab+b\beta = Q,$$
$$b\beta\{(a+b+c)(a+b+\gamma)-b^2\} = X,$$
$$c(a+\gamma) = Y,$$

we have
$$AP = Eb\gamma,$$

and
$$A'\{P.Q+X\phi\} = Eb\gamma\{Q+Y\phi\}$$
$$= AP\{Q+Y\phi\}.$$

Since ϕ is small, we have

$$A' = A\left\{1 - \frac{X - PY}{P.Q}\phi\right\}.$$

Writing $a = 0$, i.e. when the key k is open, we have

$$A'_0 = A\left\{1 - \frac{b\beta c}{P(\beta+c)}\phi\right\}.$$

Writing $a = \infty$, i.e. when the key k is closed, we have

$$\text{A}'_\infty = \text{A}\left\{1 + \frac{abc}{\text{P}(b+c)}\phi\right\}.$$

$$\text{A}'_\infty - \text{A}'_0 = \text{A}\frac{bc\phi}{(b+c)(b+\gamma)}.$$

A may be written $\text{E}\dfrac{b\gamma}{a(b+\gamma)+(b+c)\gamma}$, β being eliminated by the use of the equation $b\beta = c\gamma$.

I the intensity of the action on the galvanometer needle due to the change of the electromotive force at its terminals is measured by $(\text{A}'_\infty - \text{A}'^0)\,a^{\frac{1}{2}}$.

Hence I varies as

$$\frac{b^2 c\gamma a^{\frac{1}{2}}}{(b+c)(b+\gamma)\{(b+\gamma)a+(b+c)\gamma\}}\phi.$$

To determine the best values to give to a, c, γ, the partial differential coefficients of I are equated to zero.

$$\left.\begin{aligned}\frac{d\text{I}}{da} &= 0 \text{ if } a(b+\gamma)-(b+c)\gamma = 0\\ \frac{d\text{I}}{dc} &= 0 \text{ if } ab(b+\gamma)-(c^2-b^2)\gamma = 0\\ \frac{d\text{I}}{d\gamma} &= 0 \text{ if } a(b^2-\gamma^2)-(b+c)\gamma^2 = 0\end{aligned}\right\} \quad . \quad . \quad . \quad (a)$$

Or if either a, or c, or γ is infinite, $\dfrac{d\text{I}}{da}$, $\dfrac{d\text{I}}{dc}$, $\dfrac{d\text{I}}{d\gamma}$ are all zero.

The value of I is then equal to zero, and these solutions give minimum values to I.

The values of a, c, γ which give a maximum value to I are obtained by solving the set of equations (a) and we have

$$a = b,$$

$$c = 2b,$$

$$\gamma = \frac{b}{2},$$

$$\text{and} \quad \beta = b.$$

In making this investigation we have assumed that the electromotive force and resistance of the battery examined are

both independent of the current passing through it. This is not generally true, and may be very far from it.

Again, the currents which can be passed through certain forms of low resistance batteries without seriously disturbing the conditions assumed are so small that the values of a, β, c, γ we have determined as giving the most sensitive arrangement, provide much too low an external resistance for practical purposes. Indeed the whole of the latter part of the investigation has little more than academical interest.

APPENDIX VI.

The Electrostatic Analogue of Wheatstone's Bridge.

An arrangement of condensers analogous in electrostatics to the arrangement of resistances, called Wheatstone's bridge, in electrodynamics is of some interest.

In Wheatstone's bridge, Figure 70, four conductors, whose

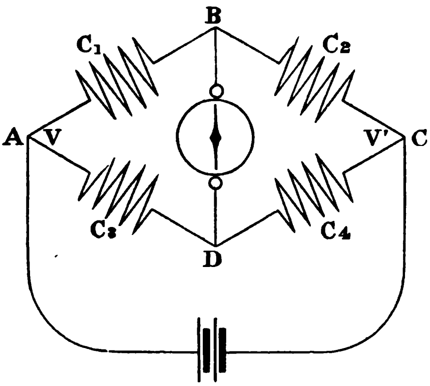


Fig. 70.

conductivities are c_1, c_2, c_3, c_4, are connected in a quadrilateral ABCD. A and C are maintained at different potentials v and v' by a battery, and the difference of the potentials of B and D is examined by means of a galvanometer connected between those points. If no current flows along BD, the potential at B is $\dfrac{c_1 v + c_2 v'}{c_1 + c_2}$, and that at D is $\dfrac{c_3 v + c_4 v'}{c_3 + c_4}$. If $c_1 c_4 = c_2 c_3$, B and D are at the same potential.

In the electrostatic analogue, Figure 71, the four conductors are replaced by four condensers whose capacities are k_1, k_2, k_3, k_4 connected in the quadrilateral *abcd*. *ac* are placed at potentials

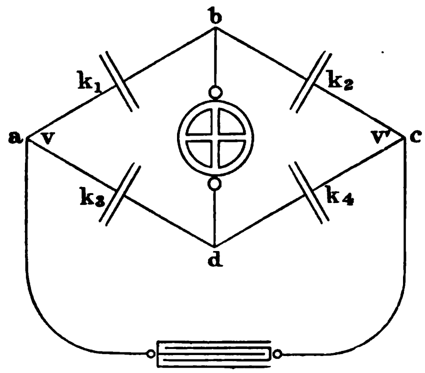


FIG. 71

vv' by connections to a charged condenser, and the difference of the potentials of *b* and *d* is examined by means of an electrometer connected between those points. If there is no displacement of electricity along *bd*, the potential at *b* is $\frac{k_1 v + k_2 v'}{k_1 + k_2}$, and that at *d* is $\frac{k_3 v + k_4 v'}{k_3 + k_4}$. If $k_1 k_4 = k_2 k_3$, *b* and *d* are at the same potential.

To apply this method to the measurement of the capacity of a condenser placed in the position say of k_1, two condensers would be required for k_3 and k_4, whose ratio is accurately known, and for k_2 another condenser finely adjustable over a large range. Such condensers are not generally found in testing rooms, and the author does not suppose that the method has been practically used.

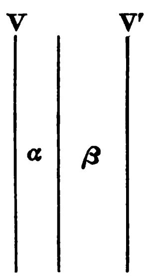


FIG. 72.

The analogue of a slide wire bridge would be a condenser shown diagrammatically in Fig. 72, with a fluid dielectric and three parallel plates, the distances between which are adjustable. If the two outer plates are placed at potentials v and v', the dis-

tances between them and the inner plate being a, β respectively, the potential of the latter will be $\frac{\beta v + a v'}{a+\beta}$ approximately: the formula may be made correct with any desired degree of accuracy by increasing the area of the plates in proportion to the distance between them. If such a three-plate condenser were used, the middle plate would be the point d of Fig. 72, and the outer plates would be connected to a and c.

A third arrangement, intermediate between those of Figs. 70 and 71, is shown in the annexed diagram (Fig. 73). One circuit

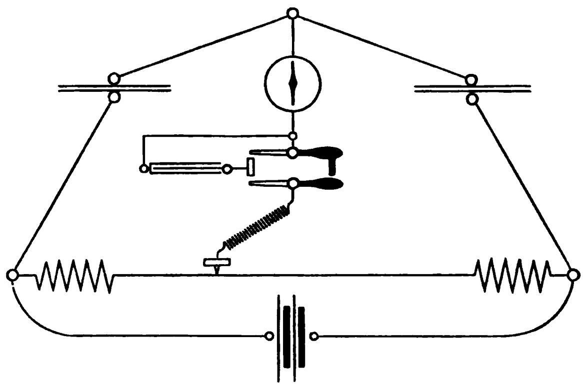


FIG. 73.

ABC between the battery terminals contains the two condensers, and the other ASC is a metre bridge or a set of slides. A battery used as a source of electromotive force keeps a current passing along the bridge circuit. This method was devised by Mr. Gott, and is in practical use for comparing submarine cables with standard condensers. For comparing the potentials at BS, Mr. Gott employs, instead of an electrometer, a galvanometer and a key, the latter of which is used to charge a subsidiary condenser from the two points. The charge passing through the galvanometer shows if there is any difference between their potentials.

THE END.